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Title No. 53-30

Proposed

Recommended Practice for Evaluation of Compression Test Results of Field Concrete*

Reported by ACI Committee 214

WILLIAM A. CORDON
Chairman

EDWARD A. ABDUN-NUR
DELMAR L. BLOEM
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HERBERT K. COOK
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SYNOPSIS

Statistical methods provide valuable tools for assessing results of strength tests, and such information is also of value in refining design criteria and specifications. The report discusses briefly the numerous variations that occur in the strength of concrete and presents statistical methods which are useful in interpreting these variations. Criteria are offered that can be used to establish specifications and maintain required uniformity.

INTRODUCTION

The primary function of compression tests of field concrete is to insure production of uniform concrete of desired strength and quality. Concrete being a hardened mass of heterogeneous materials is subject to the influence of numerous variables. Characteristics of each of the ingredients of concrete may cause variations depending upon their uniformity. Variations may also be introduced by practices used in proportioning, mixing, transporting, placing, and curing. In addition to the variations which exist in concrete itself, strength variations will also be introduced in fabrication, testing, and care of test specimens. Variations in the strength of concrete must be accepted; but consistent concrete of adequate quality can be produced with confidence if proper control is maintained, test results are properly interpreted, and limitations are considered.

*Title No. 53-30 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 28, No. 6, Dec. 1956, *Proceedings* V. 53. Separate prints are available at 50 cents each. **Discussion** (copies in **triplicate**) should reach the Institute not later than Mar. 1, 1957. Address 18263 W. McNichols Rd., Detroit 19, Mich.

This report was submitted to letter ballot of the committee which consists of 13 members, with 13 voting affirmatively. It is released by the Standards Committee for publication and discussion with view to its consideration for adoption as an Institute Standard at the 53rd Annual Convention, Dallas, Feb. 25-28, 1957.

Proper control is achieved by the use of satisfactory materials, properly mixing these materials into economical concrete of desired quality, and good practices in transporting, placing, curing, and protecting the fresh concrete. Although the complex variables of concrete preclude complete uniformity, excessive variation of concrete strength signifies inadequate concrete control. Improvement in control may permit a reduction in the cost of concrete since the average strength can be adjusted more closely to specification requirements.

Compressive strength is not necessarily the most critical factor in proportioning concrete mixes since other factors, such as durability, may impose lower water-cement ratios than are required to meet strength requirements. In such cases strength will of necessity be in excess of structural demands. Strength tests are valuable in such circumstances, however, since with established mix proportions, variations in strength are indicative of variations in other properties.

Test specimens indicate potential rather than actual strength of a structure and poor workmanship in placing and curing may cause strength reductions which are not reflected in tests. Wherever practicable, conclusions on strength of concrete should be derived from a pattern of tests from which the characteristics and uniformity of the concrete can be more accurately estimated. To place too much reliance on too few tests may result in erroneous conclusions.

Statistical methods have not been used to any great extent in the design and construction of concrete structures, except by large construction agencies. These methods provide tools of considerable value in assessing results of strength tests and such information is also of value in refining design criteria and specifications. This report discusses briefly variations that occur in the strength of concrete, presents statistical methods which are useful in interpretation of these variations, and offers criteria that can be used to establish specifications and maintain required uniformity. Although this report is written for compression tests, the statistical methods presented may also be applied to flexural strength and other tests of concrete.

VARIATIONS IN STRENGTH

The magnitude of variations in the strength of concrete test specimens depends upon how well the materials, concrete manufacture, and tests are controlled. Differences in strength can be traced to two fundamentally different sources as shown in Table 1: (a) difference in strength-producing properties of the concrete mixture, and (b) apparent difference in strength caused by discrepancies in tests.

Properties of concrete

It is well established that strength of concrete is governed to a large extent by the water-cement ratio. The first criterion for producing concrete of uniform strength, therefore, is a uniform water-cement ratio. Since the

TABLE 1—PRINCIPAL SOURCES OF STRENGTH VARIATION

Variations in the properties of concrete	Discrepancies in testing methods
Changes in water-cement ratio: Poor control of water content Excessive variation of moisture in aggregate	Inconsistent sampling procedures
Variations in water requirement: Aggregate grading Nonuniform materials	Nonuniform fabrication techniques: Amount of compaction Excessive handling of sample Care of fresh cylinders
Variations in characteristics and proportions of ingredients: Aggregates Cement Pozzolans Admixtures	Change in curing: Temperature variation Variable moisture
Variations in transporting, placing, and compaction	Poor testing procedures: Cylinder capping Compression tests
Variations in temperature and curing	

quantity of cement can be measured accurately, the problem of maintaining a uniform water-cement ratio is primarily a problem of controlling the water content. This problem is aggravated because of the variable quantity of free moisture in aggregates.

The uniformity of concrete can be no better than the uniformity of concrete aggregates, cement, and admixtures used since each will contribute to the variations of concrete strength. The temperature of fresh concrete influences its water-cement ratio for a given consistency and consequently contributes to strength variation. Construction practices may cause variations in strength due to inadequate mixing, poor compaction, delays, and improper curing.

Use of admixtures presents additional problems in maintaining uniformity of strength since each admixture adds another variable to concrete. Accelerators, retarders, pozzolans, and air-entraining agents must be carefully controlled and allowance made for their influence on strength.

Testing methods

Concrete tests may or may not include all the variations in strength of concrete in place depending on what variables are introduced after test specimens have been made. On the other hand, discrepancies in sampling, fabrication, curing, and testing of specimens may indicate variations in strength which do not exist in the concrete in place. The project is unnecessarily penalized when variations from this source are excessive. Good testing methods will reduce these variations and standard testing procedures such as those described in ASTM standards should be established and followed without variation.

The importance of using accurate compression testing machines and other laboratory equipment should need no emphasis since test results can be no more accurate than the equipment used. *Uniform test results are not necessarily accurate test results.* Laboratory equipment and procedures should be calibrated and checked periodically.

ANALYSIS OF STRENGTH DATA

To obtain maximum information, a sufficient number of field compression tests should be made to be representative of the concrete produced and appropriate statistical methods should be used to interpret the test results. Statistical methods provide the best basis for assessing from such results the potential quality and strength of the concrete in a structure and expressing results in the most useful form.

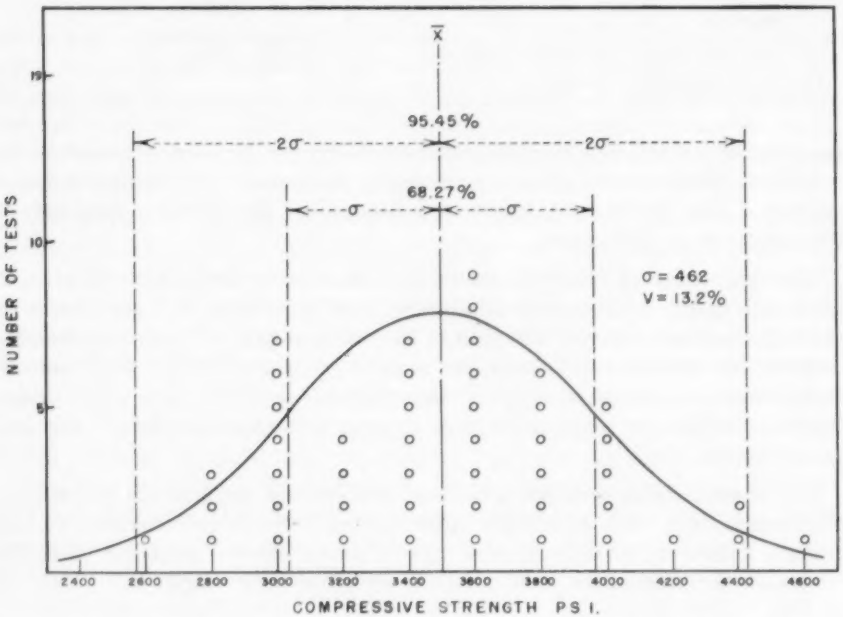

Fig. 1—Normal frequency distribution of strength data from 46 tests

Statistical functions

The strength of concrete test specimens on controlled projects can be assumed to fall into some pattern of the normal frequency distribution curve as illustrated in Fig. 1. For more complete statistical studies, determination of the actual shape of the curve may be desirable. Where there is good control the strength, values will be bunched close to the average, and the curve will be tall and narrow. As the variations in strength increase, the values spread and the curve becomes low and elongated (Fig. 2). Certain functions of the normal frequency curve are useful in evaluating strength data:

Average, $\overline{X}$ — The average strength of all cylinders.

$$\overline{X} = \frac{X_1 + X_2 + X_3 + \ldots + X_n}{n} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

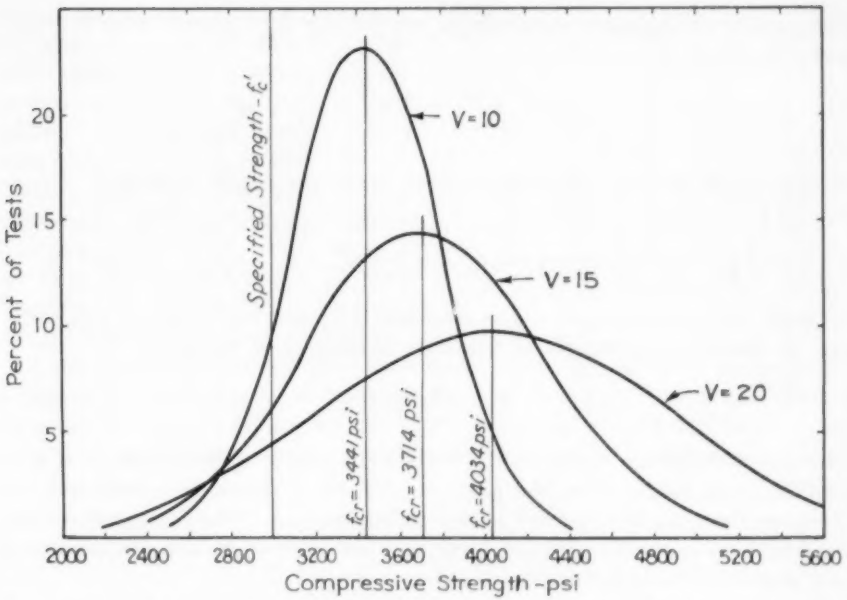

Fig. 2—Normal frequency curves for coefficients of variation of 10, 15, and 20 percent

Required average strength f_{cr} based on a probability of one in ten that a test will fall below a specified strength, f_c' of 3000 psi

Where X_1, X_2, X_3 . . . X_n are the strength results of individual specimens and n is the total number of specimens tested.

Standard deviation, σ—The most generally recognized measure of dispersion is the root-mean-square deviation of the strengths from their average. Referring to Fig. 1 the standard deviation is the radius of gyration of the area under the theoretical probability curve about the center. The standard deviation is found by extracting the square root of the average of the squares of deviations of individual strengths from their average.*

$$\sigma = \sqrt{\frac{(X_1 - \overline{X})^2 + (X_2 - \overline{X})^2 + \ldots + (X_n - \overline{X})^2}{n}} \quad \ldots\ldots\ldots (2)$$

For ease of computation, when a calculating machine is available, standard deviation can be obtained by dividing the sum of the squares of individual

*Some texts on statistics refer to σ as the standard deviation of the universe (infinite number of tests) and s as the standard deviation of a sample. In computing s instead of σ in Eq. (2), n is replaced by $(n - 1)$. This practice was not followed in the manual of ASTM Committee E-11 on Quality Control of Materials (*ASTM Special Technical Publication* No. 15-C) and is significant only when the number of samples is small (less than 30). Generally, control of concrete uniformity involves a large number of samples collected over a period of time and the difference between methods is negligible. On the other hand, the number of samples is usually small (two or three tests) in computing the within-test standard deviation σ_1. Eq. (5) is useful in this case since the factor $1/d_2$ and a reliable value of $\overline{R}$ produce a true or unbiased estimate of σ_1 as distinct from observed values as in Eq. (2), (3), and (7). The values of Table 2 were established from a large number of tests on the basis of the formulas as presented.

strengths by the numbers of cylinders, subtracting the square of their average, and extracting the square root.

$$\sigma = \sqrt{\frac{X_1^2 + X_2^2 + X_3^2 + \ldots + X_n^2}{n} - \bar{X}^2} \quad \ldots\ldots (3)$$

or rearranged to bring the fraction from under the square root sign:

$$\sigma = \frac{1}{n}\sqrt{n(X_1^2 + X_2^2 + X_3^2 + \ldots + X_n^2) - (X_1 + X_2 + X_3 + \ldots + X_n)^2} \quad (3a)$$

Coefficient of variation, V—The standard deviation expressed as a percentage of the average strength is called the coefficient of variation.

$$V = \frac{\sigma}{\bar{X}} \times 100 \quad \ldots\ldots (4)$$

This function makes it possible to express the degree of dispersion on a percentage basis rather than the absolute. Table 2 shows the coefficients of variation that can be expected on controlled projects. The ratings of control are based on experience from a large number of projects and are presented as a general guide in evaluation of concrete control.

TABLE 2—STANDARDS OF CONCRETE CONTROL

Class of operation	Coefficients of variation for different control standards			
	Excellent	Good	Fair	Poor
Over-all variations:				
General construction	Below 10.0	10.0 to 15.0	15.0 to 20.0	Above 20.0
Laboratory control	Below 5.0	5.0 to 7.0	7.0 to 10.0	Above 10.0
Within-test variations:				
Field control	Below 4.0	4.0 to 5.0	5.0 to 6.0	Above 6.0
Laboratory control	Below 3.0	3.0 to 4.0	4.0 to 5.0	Above 5.0

Note: These standards represent the average for 28-day cylinders computed from a large number of tests. Different values for other than average concretes can be expected.

Range, R—Range is found by subtracting the lowest strength in any group of tests from the highest strength. Range is useful in computing the within-test standard deviation discussed in the following section.

Batch variations

As mentioned previously, variations in results of strength tests can be traced to two different sources: (a) properties of the concrete mixture, and (b) discrepancies in testing methods. It is possible by analysis of variance to compute the variations attributable to each source.

Within-test variation—The variation in strength of concrete within a single batch can be found by computing the variations of a group of cylinders fabricated from that batch. It is also convenient to assume that a test

sample of concrete is uniform and any variation between companion cylinders fabricated from a given sample is caused by fabricating, curing, and testing discrepancies. Samples taken from different parts of a batch will include variations due to mixer inefficiency. Companion specimens fabricated from samples from different locations in the mixer can be used therefore to differentiate between mixer efficiency and testing efficiency.

TABLE 3—FACTORS FOR COMPUTING "WITHIN TEST" STANDARD DEVIATION*

Number of specimens	d_2	$1/d_2$
2	1.128	0.8865
3	1.693	0.5907
4	2.059	0.4857
5	2.326	0.4299
6	2.534	0.3946
7	2.704	0.3698
8	2.847	0.3512
9	2.970	0.3367
10	3.078	0.3249

*From Table B2, "Manual on Quality Control of Materials," *ASTM Special Technical Publication* No. 15-C.

A single batch of concrete provides meager data for statistical analysis and companion cylinders from at least ten samples of concrete are recommended to establish reliable values for $\overline{R}$. The within-test standard deviation and coefficient of variation can be conveniently computed as follows:

$$\sigma_1 = \frac{1}{d_2}\overline{R} \quad \ldots\ldots(5)$$

$$V_1 = \frac{\sigma_1}{\overline{X}} \times 100 \quad \ldots\ldots(6)$$

where

σ_1 = within-test standard deviation

$\frac{1}{d_2}$ = a constant depending upon the number of cylinders in each group (Table 3)

$\overline{R}$ = average range of groups of companion cylinders

V_1 = within-test coefficient of variation

$\overline{X}$ = average strength

Cumulative distribution

If the theory of the "weakest link" is applied to concrete, the number of tests lower than the desired strength is more important in computing the load-carrying capacity of concrete structures than the average strength obtained. It is impractical, however, to specify minimum strength since the law of normal probability indicates we can expect one strength out of every six tests to be lower than the standard deviation (σ) below average ($\overline{X}$), one out of every 44 will be lower than 2σ below average, and one out of 741 will be lower than 3σ below average.

The cumulative distribution curve is plotted by cumulating the number of tests below any given strength. Fig. 3 shows a group of cumulative distribution curves based on percentage and plotted as straight lines on the probability scale. This chart provides a convenient means of securing cumulative probability data when the coefficients of variation are known.

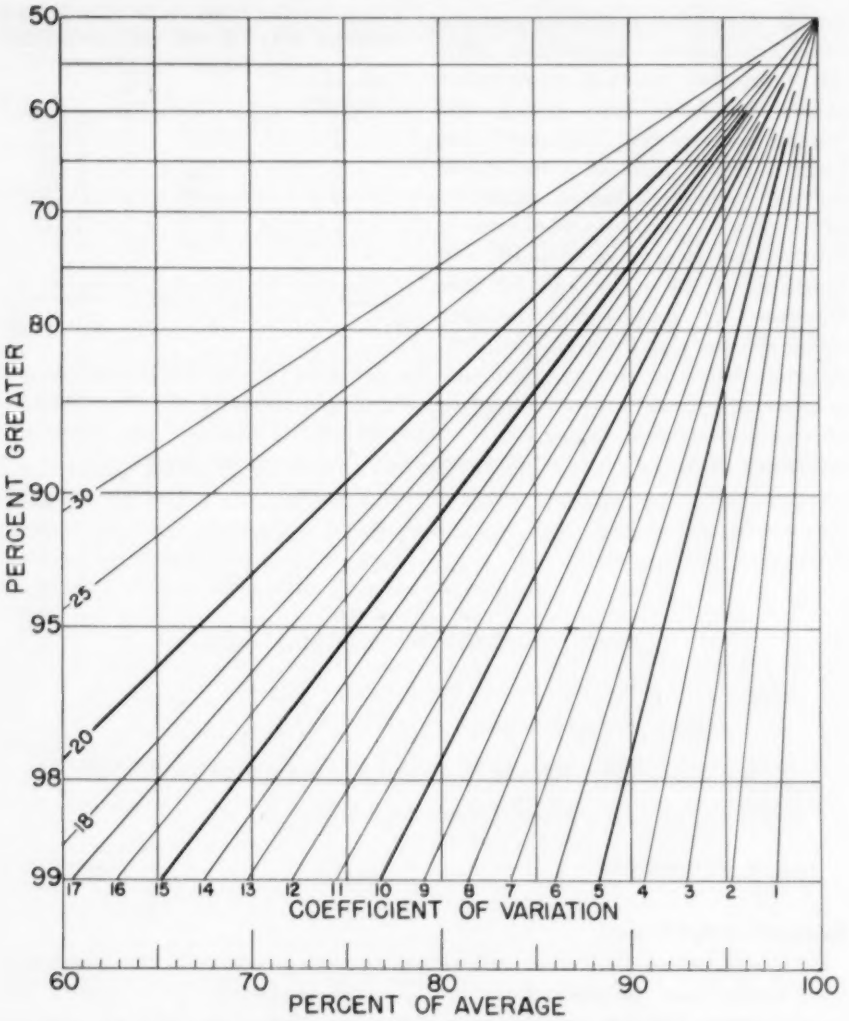

Fig. 3—Cumulative distribution curves for different coefficients of variation

Quality-control charts

Quality-control charts have been used by manufacturing industries for many years as an aid to uniformity and efficiency in production. Methods are well established for the setting up of such charts and are outlined in convenient form in the ASTM "Manual on Quality Control of Materials." Based on the pattern of previous results and limits established therefrom, trends become apparent as soon as new results are plotted. Points which fall outside the calculated limits are indicative that something has affected the

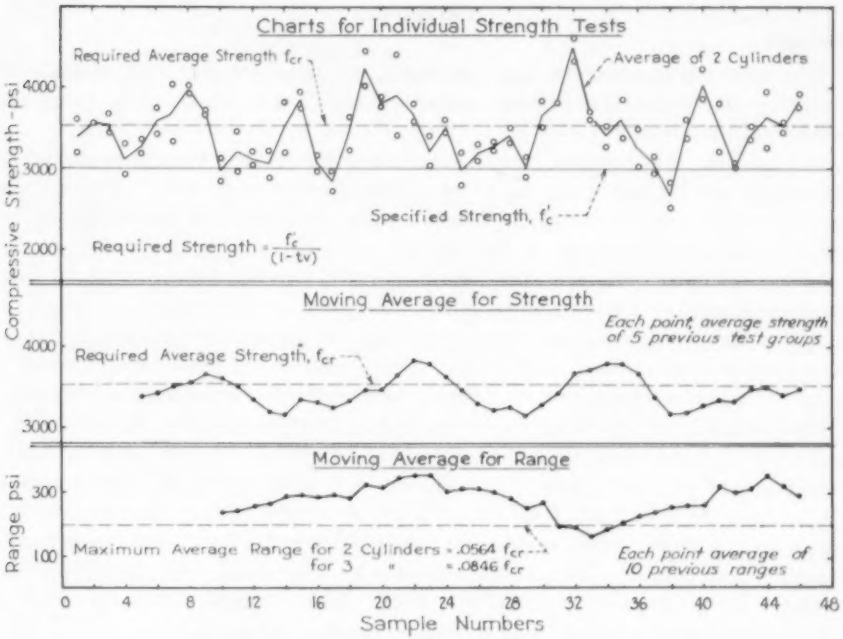

Fig. 4—Quality control charts for concrete

control of the process. Such charts are recommended wherever concrete is in continuous production over considerable periods.

Three simplified charts prepared specifically for concrete control are illustrated in Fig. 4. (As pointed out in Example 4, p. 578, this is not particularly good control.) While these do not contain all the features of formal control charts they should prove useful to the engineer, architect, and plant superintendent.

(a) A chart in which the results of all strength tests are plotted as received. The line for the required average strength is established as indicated in the section on criteria, p. 572, and the specified design strength shows the number of low tests.

(b) Moving average for compressive strength where the average is plotted for the previous five sets of two companion cylinders for each day or shift and the specified strength in this case is the lower limit. This chart is valuable in indicating trends and will show the influence of seasonal changes, changes in materials, etc. The number of tests averaged to plot moving averages with an appropriate lower limit (Table 5) can be varied to suit each job.

(c) Moving average for range where the average range of the previous ten groups of companion cylinders is plotted each day or shift. The maximum average range allowable for good laboratory control is also plotted. Maximum average range is determined as discussed in the section on criteria.

Fig. 4 shows charts a, b, and c for 46 tests. To be fully effective charts should be maintained throughout the entire job.

CRITERIA

Strength

The strength of concrete test specimens and the load carrying capacity of concrete structures have been correlated by tests made with full-scale structural members.

Indications of structural strength are also obtained by testing prototypes, by coring, and by dynamic and other nondestructive type tests. These tests are not widely used, however, and the strength of control cylinders is generally the only tangible evidence of the quality of concrete in a structure. Compressive strength data are therefore important in establishing criteria, but their limitations must be considered. Because of the possible disparity between the strength of test specimens and the load carrying capacity of a structure it is dangerous to place too much reliance on inadequate strength data. It is also wrong to conclude that the strength of a structure is in jeopardy when a single test fails to meet specified strength requirements. As previously indicated, random variations and occasional failures to comply with strength requirements are inevitable. Accordingly inflexible strength requirements are unrealistic and control of the pattern of results rather than individual values is the most appropriate basis for both specifications and the general assessment of results. This is the réason why statistical concepts have such potential value in concrete control. From these concepts there have been developed tools to measure and indices to control variation.

The consequences of a localized zone of low-strength concrete in a structure depend on many factors; included are the probability of early overload, the location and magnitude of the low quality zone in the structural unit, the degree of reliance placed on compressive strength in design, the initial cause of the low strength, and the consequences, economically and otherwise, of structural failure. As a general guide it is the opinion of the committee that a reasonable control of structural concrete would be provided if the probability of any test falling below f_c' used in design is no greater than 0.10 (1 in 10). This tolerance of test failure does not imply acceptance of consecutive failures in 10 percent of the structure, but must be expressed as a continuous control rather than an over-all percentage. Greater probability of low-strength specimens is allowable in other classes of concrete (Table 5) but the final criterion adopted is obviously a matter for the designer's decision based on his intimate knowledge of the conditions that are likely to prevail.

To satisfy strength performance requirements expressed in this fashion the average strength of concrete must obviously be in excess of f_c', the degree of excess strength depending on the expected uniformity of concrete production and the allowable proportion of low tests. The required average strength, f_{cr}, for any design can be approximated directly from Fig. 5 or computed as follows:

$$f_{cr} = \frac{f_c'}{(1 - tV)} \quad \text{.....(7)*}$$

*The required average modulus of rupture for beams can be obtained using Eq. (7) by substituting the specified modulus of rupture (SMR) for f_c' and using an appropriate coefficient of variation for flexure tests.

where

f_{cr} = required average strength
f_c' = design strength specified
t = a constant depending upon the proportion of tests that may fall below f_c' and the number of samples used to establish V (Table 4)
V = forecasted value of the coefficient of variation expressed as a fraction

Table 4 indicates a value for t of 1.282 for a probability of 1 in 10 of a test being low, and Table 2 indicates that for good field control the coefficient of variation should not be greater than 15 percent. In this case f_{cr} is equal to 1.24 f_c' and for a structural design strength of 3000 psi the mixture will be proportioned for an average strength of 3720 psi. This value of f_{cr} could be reduced with better control and must be increased for poorer control as illustrated in Fig. 2. Where a batch of low-strength concrete is not so critical

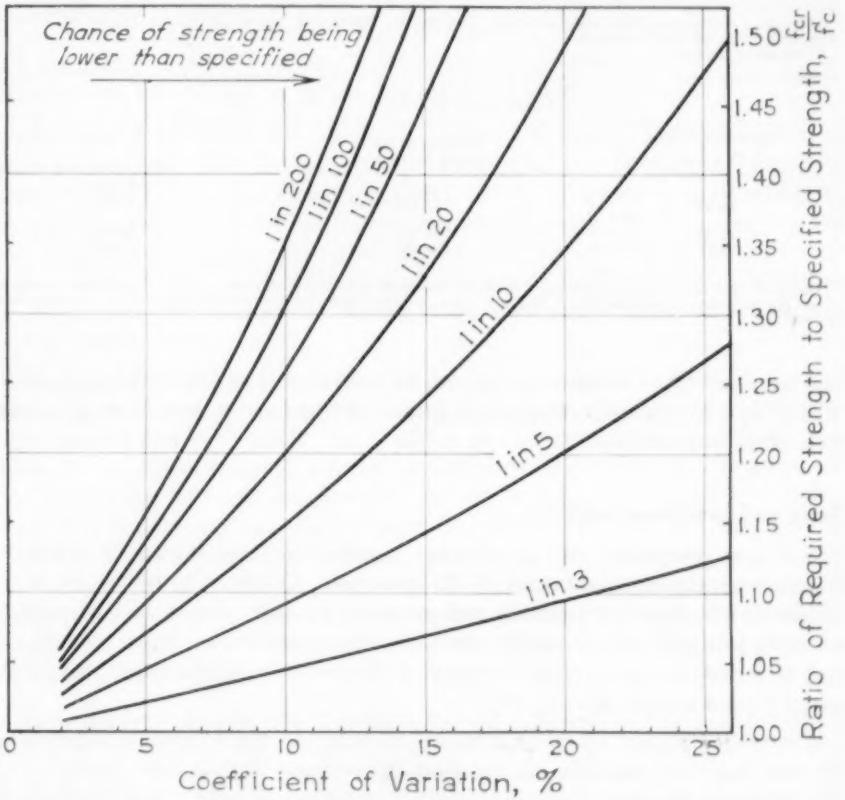

Fig. 5—Ratio of required average strength f_{cr} to specified strength f_c' for various coefficients of variation and chances of falling below specified strength

This chart is based on the distribution of the normal variate while Eq. (7) is based on the distribution of "Student's t." For a small number of tests Eq. (7) requires a higher, hence a more reliable value for f_{cr}

TABLE 4—VALUES OF t*

No. of samples minus 1†	Percentage of tests falling within the limits $\bar{X} \pm t\sigma$							
	50	60	70	80	90	95	98	99
	Chances of falling below lower limit							
	2.5 in 10	2 in 10	1.5 in 10	1 in 10	1 in 20	1 in 40	1 in 100	1 in 200
1	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657
2	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925
3	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841
4	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604
5	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032
6	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707
7	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499
8	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355
9	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250
10	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169
15	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947
20	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845
25	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787
30	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750
∞	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576

*Values of t extracted from table originally produced by Fisher and Yates, "Statistical Tables for Biological Agriculture and Medical Research."
†Degrees of freedom.

Other values of t for $n - 1 = \infty$

Percentage within $\bar{X} \pm t\sigma$	Chances of falling below lower limit	t
40	3 in 10	0.524
68.27	1 in 6.3	1.000
95.45	1 in 44	2.000
99.73	1 in 741	3.000

Values of t increase for small samples due to the unreliability of small samples to establish a true estimate of σ. The advantage of establishing V in Eq. (7) from a large number of tests becomes apparent in the reduction of t and f_{cr}.

and two low test cylinders in ten can be tolerated, t equals 0.842 and $f_{cr} = 1.14\, f_c'$ or 3430 psi for a design strength of 3000 psi and a coefficient of variation of 15 percent.

Tests and specimens required

For any particular job, a sufficient number of tests should be made to insure accurate representation of the concrete. Concrete tests can be made either on the basis of time elapsed or cubic yardage placed and conditions on each job will determine the most practical number of tests needed. A test is defined as the average strength of all specimens of the same age fabricated from a sample of concrete.

Centralized project—A project where all concrete operations are supervised by one engineer provides an excellent opportunity for uniform control and for accurate estimates of reliability with a minimum of tests. After operations are progressing smoothly, tests taken each day or shift, depending upon the volume of concrete produced, are sufficient to obtain data which are representative of all structures. Under adequate control of all operations, it is unnecessary to test the concrete at the point of placement unless dictated

by unusual problems. In general, it is advisable to make a sufficient number of tests so that each different type of concrete placed during any one day will be represented by two standard 6 x 12-in. cylinders to be tested at 28 days. Single specimens taken from two different batches each day will provide more reliable information on over-all variations, but it is usually desirable to make companion specimens from the same sample to obtain a check on the within-test variation.

Producer—The number of control tests required to properly evaluate the operations of a concrete mixing plant is similar to that of a centralized project since operations and procedures are uniform even though concrete may be supplied to many different structures. Problems encountered and control desired will dictate the number of tests for a given mixing plant but if operations are progressing uniformly, reliable information on over-all plant operation over a period can be obtained with one 28-day cylinder each day or each shift for each different type of concrete. If information regarding the accuracy of tests is desired, companion specimens will be required to differentiate the variations due to testing from plant variations.

Consumer—The consumer or owner, usually represented by an architect or engineer, has more than academic interest in the strength of concrete since the safety and load carrying capacity of the structure may be impaired by poor control and batches of low-strength concrete. If all operations are controlled by one engineer, the problem is simplified as indicated for a centralized project. The popular practice of purchasing concrete from a separate producer and contracting the testing work separately complicates concrete testing and control.

The consumer is not only interested in the concrete as it leaves the plant but is interested in every operation until the concrete is placed in the structure and adequately cured. In addition to mixing-plant-control tests, the engineer (architect) should require performance tests to insure concrete of desired properties. Performance tests should be taken at the point of placement and companion specimens should be made for greater accuracy and to establish responsibility for variation between the producer and the testing laboratory. The contractor may also contribute to variations with poor construction practices, hence correlation between control tests taken at the producer's plant and performance tests at the point of placement will help establish responsibility.

The number of specimens required by the engineer (architect) should be based on established standards but may be reduced as the reliability of the producer, the laboratory, and the contractor are established.

Testing laboratory—The testing laboratory has the responsibility of making accurate tests, and concrete will be penalized unnecessarily if tests show greater variations than actually exist. Since the range between companion specimens from the same sample can be assumed to be the responsibility of

the testing laboratory, a control chart for ranges (Fig. 4) should be maintained by the laboratory as a check on the uniformity of its operations. The range between companion cylinders depends on the number of specimens in the group and the within-test variation. This relationship is expressed by the following equation:

$$\bar{R}_m = f_{cr} V_1 d_2 \quad (8)$$

The within-test coefficient of variation V_1 should not be greater than 5 percent for good control (Table 2), and the estimate of the corresponding average range will be:

$\bar{R}_m = (0.05 \times 1.128) f_{cr} = 0.05640 f_{cr}$ for groups of two companion cylinders
$\bar{R}_m = (0.05 \times 1.693) f_{cr} = 0.08465 f_{cr}$ for groups of three companion cylinders
$\bar{R}_m$ becomes the maximum for average range in control chart (c) of Fig. 4.

Type of specimens required—A cylinder of representative concrete 6 in. in diameter and 12 in. high which has been moist cured for 28 days is generally recognized as a standard specimen for strength and uniformity of concrete if the coarse aggregate does not exceed 2 in. in nominal size. Many times, particularly in the early stages of a job, it becomes necessary to estimate the strength of concrete being produced before the 28-day strength results are available. Concrete cylinders from the same batch should be made and tested at 7 days or earlier ages if desired. The 28-day strength can be estimated by extrapolating early test data.

The strength of concretes at later ages, particularly where pozzolans or cements of slow strength gain are used, is more realistic than the standard 28-day strength. Some structures will not be loaded until concrete has been allowed to cure for longer periods and advantage can be taken of strength gain after 28 days. Some concretes have been found to produce less than 50 percent of their ultimate strength at 28 days. If design is based on strength at later ages, it becomes necessary to correlate these strengths with standard 28-day cylinders since it is not practicable to use later age specimens for concrete control. If possible the correlation should be established by laboratory tests before construction starts. If mixing plants are located in one place for long enough periods, it is advisable to establish this correlation for reference even though later age concrete is not immediately involved.

Curing concrete test specimens at the construction site and under job conditions is sometimes recommended since it is more representative of the curing given the structure; these special tests should not be confused with, nor replace, standard control tests. Tests of job-cured specimens may be highly desirable and are necessary when determining the time of form removal, particularly in cold weather, and when establishing the strength of steam-cured concrete pipe, block, and structural members.

The potential strength and uniformity of concrete can be established by standard 6 x 12-in. cylinders made and cured under standard conditions. Strength specimens of concrete made or cured under other than standard

conditions provide additional information but should be analyzed and reported separately.

Rejection of doubtful specimens—The practice of arbitrary rejection of test cylinders which appear "too far out of line" is not recommended since the normal pattern of probability establishes the possibility of such results. Discarding tests indiscriminately could seriously distort the strength distribution, making analysis of results less reliable.

It occasionally happens that one of a set of strength values deviates so far from the mean as to be highly improbable. The committee recommends that a specimen from a test of three or more specimens be discarded if its deviation from a test mean is greater than 3 σ and should be accepted with suspicion if its deviation is greater than 2 σ. If questionable variations have been observed during fabrication, curing, or testing of such a specimen, it should be rejected. The test average should be computed from the remaining specimens.

A test (average of all specimens of a sample) should never be rejected unless the specimens are known to be faulty, since it represents the best available estimate for the sample. Rejection of low tests would nullify strength specifications limits shown in Table 5.

SPECIFICATIONS FOR CONCRETE STRENGTH

Based on the foregoing considerations, Table 5 is presented as a guide in establishing strength specifications. The limits in each case are based on a

TABLE 5—GUIDE FOR CONCRETE STRENGTH SPECIFICATIONS

Class of concrete	Probability of tests falling below specified strength	Requirements for averages of consecutive tests	Allowable number of consecutive tests below the specified strength	
			Questionable	Not acceptable
Structural				
Ultimate strength design	1 in 20	Average of 3 above f_c'	1	2
General	1 in 10	Average of 5 above f_c'	2	3
	1 in 10	Average of 3 above 0.93 f_c'	2	3
Pavements				
Compressive strength	2 in 10	Average of 3 above 0.86 f_c'	3	4
Flexural strength	2 in 10	Average of 3 above 0.86 specified modulus of rupture	3	4
Dams and irrigation works	2 in 10	Average of 3 above 0.86 f_c'	3	4
Miscellaneous construction where strength is less critical	3 in 10	Average of 3 above 0.82 f_c'	4	5

These suggested limits are based on the premise that concrete is proportioned to produce an average strength equal to the required strength [Eq. (7)], with a coefficient of variation of 15 percent.

Consecutive low strength tests should be accepted with suspicion. In the questionable column of Table 5 the probability of obtaining the indicated number of consecutive low tests is about one to five in 100, which does not justify rejection but at the same time indicates a possibility that these tests may be caused by an unexpected drop in the average strength or that production is not in control.

probability of rejection of about 2 in 1000, with an average strength equal to f_{cr}. Failure to meet these limits will be an indication that the average strength does not equal f_{cr}. This could be caused by lower strength or poorer control than expected, or both. In either case the average strength should be increased.

The requirements for ultimate strength design are more restrictive and require closer control since load factors are selected on the premise that the strength of concrete in a structure will not be less than the assumed value of f_c' and do not allow for the variability of concrete strength.

Most specifications for concrete strength require that a test be comprised of three specimens from the same sample of concrete. Three specimens are necessary to obtain a reliable average for a given sample but it should also be emphasized that more reliable strength data are obtained from three tests of one cylinder each than from one test of three cylinders.

EXAMPLES

The mechanics of analyzing concrete strength data can be explained in more detail by using hypothetical examples of typical field jobs.

Example 1

Table 6 lists concrete strength data taken at a commercial mixing plant over a period of 2 months. It is desired to determine the degree of uniformity being obtained in comparison with general practice and to establish average

TABLE 6—EVALUATION OF UNIFORMITY OF PRODUCTION

Test No.	28-day strength, psi		Range, psi	Test No.	28-day strength, psi		Range, psi
	Cylinder 1	Cylinder 2			Cylinder 1	Cylinder 2	
1	3600	3190	410	24	3410	3500	90
2	3560	3550	10	25	2780	3180	300
3	3420	3670	250	26	3090	3300	210
4	2920	3300	380	27	3210	3310	100
5	3330	3190	140	28	3490	3300	190
6	3750	3400	350	29	2890	3040	150
7	4030	3920	110	30	3810	3500	310
8	4010	3540	470	31	3830	3800	30
9	3690	3650	40	32	4590	4370	220
10	2830	3120	190	33	3600	3700	100
11	2950	3450	500	34	3510	3260	250
12	3040	3200	160	35	3350	3840	490
13	3210	2900	310	36	3020	3480	460
14	3810	3190	620	37	3150	2940	210
15	3950	3730	220	38	2510	2820	310
16	3150	2960	190	39	3370	3600	230
17	2930	2700	230	40	4210	3860	350
18	3610	3210	400	41	3800	3200	600
19	4440	4000	440	42	3000	3050	50
20	3870	3770	100	43	3530	3340	190
21	4390	3600	790	44	3950	3250	700
22	3560	3790	230	45	3590	3440	150
23	3040	3400	360	46	3910	3760	150

n = 92
ΣX = 317,960
$\Sigma X/n$ = 3456 psi
ΣX^2 = 1,114,248,800
$\Sigma X^2/n$ = 12,111,400
$\bar{X}^2$ = 11,943,936

σ^2 = 167,464
σ = 409
V = 11.8 percent
$\bar{R}$ = 277
σ_1 = 277 × 0.8865 = 246
V_1 = 7.1 percent

strengths required by this plant to meet various specifications. Companion specimens have also been taken to evaluate uniformity of testing.

The computations listed with Table 6 show that an average strength of 3456 psi was obtained during the 2-month period.

The standard deviation (409 psi) was computed by taking average of the squares of the strengths, subtracting the average strength squared, and extracting the square root [Eq. (3)]. The strengths and the squares of the strengths can be added on most calculators in one operation.

The coefficient of variation (11.8 percent) is found by computing the standard deviation as a percentage of the average strength. Table 2 indicates that a coefficient of variation of 11.8 percent for average concrete indidates good uniformity.

With an average strength of 3456 psi and a coefficient of variation of 11.8 percent, Fig. 3 shows that nine tests out of ten can be expected to be greater than 85 percent of average or $0.85 \times 3456 = 2940$ psi.

The concrete plant plans to bid on furnishing concrete for two projects: the foundations for a housing project and prestressed concrete for a railroad bridge. The architects for the housing project have specified 2000-psi concrete and will permit a probability of three in ten for tests to fall below that figure. Until sufficient tests have been made to establish a coefficient of variation for this class concrete, 11.8 percent is the best estimate available. From Eq. (7) we find that for an expected coefficient of variation of 11.8 percent, and a value of t from Table 4 of 0.524, the required strength equals

$$f_{cr} = \frac{f_c'}{(1 - tV)} = \frac{2000}{(1 - 0.524 \times 0.118)} = 2130 \text{ psi}$$

The railroad has specified 4000-psi concrete with a probability of no more than one test in ten allowed to fall below that figure. The required strength in this case will equal

$$f_{cr} = \frac{4000}{(1 - 1.282 \times 0.118)} = 4710 \text{ psi}$$

Previous strength data from field tests or trial mixes will show water-cement ratios and cement factors required to produce required strengths.

The within-test coefficient of variation (7.1 percent) is computed by multiplying the average range $\bar{R}$ by the factor $1/d_2$ for two companion cylinders, dividing by the average strength, and multiplying by 100. Table 2 indicates that a coefficient of 7.1 percent denotes poor testing practices.

Example 2

To emphasize the difficulties of the consumer, assume that during construction of the bridge, performance tests made periodically at the job site

showed considerably more variation than was anticipated from the mixing plant data. Investigation revealed that the contractor's foreman at times added water to the concrete to facilitate placing operations. Since variation in water-cement ratio is one of the greatest sources of variation of concrete strength, adding water to the plant-mixed concrete increased the coefficient of variation from 11.8 to 19.5 percent, which is listed as fair control in Table 2, and the required strength f_{cr} must be increased to meet design requirements.

$$f_{cr} = \frac{4000}{(1 - 1.282 \times 0.195)} = 5330 \text{ psi}$$

This illustrates the cost of permitting poor control practices since concrete with good control as shown in Example 1 would serve the purpose equally well, with an average strength of 4710 psi instead of 5330 psi.

Example 3

A laboratory desired to check the uniformity of its operations. Table 7 represents test data taken to make this evaluation. Regular test samples were taken in the field and two companion cylinders made from each sample. The average range between companion cylinders (135 psi) and the average strength of all tests (3450 psi) are computed as shown below Table 7. The standard deviation (120 psi) is simply the average range multiplied by the constant established for two cylinders [Eq. (5)]. The coefficient of variation (3.5 percent, within-test) is the standard deviation expressed as a percentage of the average strength. Table 2 indicates that a coefficient of variation of 3.5 percent is indicative of excellent control for field testing procedures and is good for concrete mixed under laboratory controlled conditions. If the equipment and testing methods are accurate the laboratory will provide reliable data on the strength of concrete.

TABLE 7—EVALUATION OF TESTING METHODS

Sample No.	28-day strength, psi		Range, psi	Sample No.	28-day strength, psi		Range, psi
	Cylinder 1	Cylinder 2			Cylinder 1	Cylinder 2	
1	3540	3630	90	10	2990	3260	270
2	3410	3600	190	11	3040	2990	50
3	3420	3450	30	12	3210	3070	140
4	2920	3210	290	13	3810	3770	40
5	3330	3180	150	14	3950	4140	190
6	3730	3560	170	15	3150	3280	130
7	4020	3890	130	16	2950	2970	20
8	3650	3580	70	17	3610	3430	180
9	2830	3030	200	18	3870	3780	90

$\Sigma X = 124{,}150$
$\Sigma R = 2430$
$\bar{X} = \Sigma X/n = 3449$

$\bar{R} = \Sigma R/n = 135$
$\sigma_1 = 0.8865 \times 135 = 120$
$V_1 = (120 \times 100)/3449 = 3.5$ percent

Example 4

Examination of the quality control charts (Fig. 4) shows that even though the average strength is slightly lower than that required the concrete meets

suggested limits for structural concrete. Some cylinders fall below f_c', but nearly all are close to that value as would be expected from Fig. 1. This gives more confidence in the specified design strength f_c' even though some tests are lower. In addition, the large number of tests whose strengths are considerably in excess of f_c' should also be noted since there may be some compensation in load distribution for a possible low strength test.

The moving average for strength did not fall below the lower limit but whenever the moving average fell below the required average, the number of low strengths for that period increased.

The moving average for range should not vary significantly above the maximum average ($\overline{R}_m$) or the testing must be considered poor.

The control charts indicate the following:

1. The W/C ratio should be lowered to increase the average strength or if possible the coefficient of variation should be reduced to lower the required strength.

2. Control in general is good but the strength varies from high to low as shown by the moving average. An effort should be made to trace the causes of these cycles.

3. The uniformity of testing should be improved to bring the average range down to the allowable average.

For such discussion of this paper as may develop please see June 1957 JOURNAL.

Mackinac Bridge Pier Construction*

By R. E. DAVIS Jr.,† and C. E. HALTENHOFF‡

SYNOPSIS

Construction of the Mackinac Bridge, which will link the upper and lower peninsulas of Michigan, utilized the prepacked method for placing over 400,000 cu yd of concrete in the substructure.

Coarse aggregate for the concrete was placed directly in the cofferdams or caissons by self-unloading ships at rates of up to 2500 tons per hr. The voids in the coarse aggregate were intruded with heavily sanded grout which was pumped from a floating mixing plant. By such grouting, prepacked concrete, containing as little as 2.4 sacks of cement per cu yd for a compressive strength of 3000 psi, was produced at a maximum of 6250 cu yd per day.

INTRODUCTION

Completion of the new four-lane toll bridge across the Straits of Mackinac will provide a physical link between the two peninsulas of the State of Michigan. Costing $99,800,000, the project has an over-all length of 5 miles. One portion is a suspension bridge for which the distance between anchorages is 8614 ft, making it the world's longest. The center suspension span of 3800 ft is second only to the Golden Gate Bridge.

Physical difficulties imposed by severe weather conditions, a short construction season, the length of water crossing, and the depth of water have been a challenge to engineers and contractors. With the completion of all the foundation piers to above-water level this challenge has been largely met. It is anticipated that the superstructure will be completed and the bridge will be open to traffic in the fall of 1957.

All concrete for the 34 foundation piers, which will carry the 75,000-ton load of the superstructure, was placed by the prepacked method, since it was determined that this method possessed certain advantages over conventional concrete with regard to (1) physical properties, (2) plant cost, (3) ease of production and placement, and (4) reduction in the cost of materials. The total quantity of concrete in the substructure is 440,000 cu yd, of which 350,000 cu yd was placed under water at depths as great as 200 ft.

Prepacked concrete is manufactured by preplacing in the forms coarse aggregate of about ½-in. minimum size and then filling the voids of the coarse aggregate mass with a special grout composed of portland cement, fly ash,

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†Member American Concrete Institute, Assistant to Executive Vice-President, The Prepakt Concrete Co., Cleveland, Ohio.

‡Member American Concrete Institute, Manager, Mackinac Straits Bridge, St. Ignace, Mich.

sand (substantially all of which passes the No. 16 sieve), and a small quantity of intrusion aid which acts to reduce the water requirement, helps maintain the solids in suspension, and produces a small expansion prior to the time of setting. Sufficient water is employed to produce a grout of thick cream consistency, which can be readily injected into the aggregate mass to fill the voids, yet which prior to the time of setting will not bleed, thus preventing the occurrence of water voids beneath the pieces of coarse aggregate. When the intrusion aid is used in the usual amounts, it retards the time of setting, and prepacked concretes characteristically exhibit lower strengths at early ages than do corresponding conventional concretes, although they continue to gain in strength for a much longer period.

GENERAL DESCRIPTION OF WORK

The Mackinac Bridge will connect St. Ignace on the northern side of the straits with Mackinac City on the southern side. The bridge will carry a 48-ft roadway, accommodating four lanes of traffic. A clearance of 148 ft at the center of the main suspension span will permit passage underneath the bridge of the largest ships on the Great Lakes.

Two main cables, each 24¼ in. in diameter and 68 ft apart, will support the suspension span. The cable sag is 350 ft, equivalent to 1/11 of the length of the center span. Two 2¼-in. diameter wire rope suspenders, hanging from the main cables at 39-ft intervals, will support the suspended trusses.

Fig. 1 is an elevation of the bridge. The superstructure will be supported by 32 water-based and two land-based piers. The two tower piers, supporting the suspension cables on either side of the main span, are 116 ft in diameter and were founded approximately 200 ft below the surface. Each contains approximately 80,000 cu yd of concrete. The anchorage piers, in which the ends of the suspension cables will be embedded, are 135 ft long by 115 ft wide and extend to bedrock approximately 90 ft below the surface. Each contains approximately 90,000 cu yd of concrete.

The height of the piers above water ranges from 16 ft for those nearest shore to 118½ ft for the anchorages. The upper portions of all piers are reinforced. Of the 34 piers, 26 were founded directly on bedrock and eight were carried by heavy H-piles driven through the overburden which in some cases was 100 ft thick. The base of these eight piers was 10 ft below mudline, and the piling extended upward into the piers to an elevation at least 10 ft above mudline. Thirty-one of the piers were constructed by the steel sheet-pile cofferdam method, and three of the deepest piers (the two tower piers and the southern cable-support pier) were built by the open-dredge type caisson method.

Cofferdam construction

The design and dimensions of the cofferdams varied with depth of water and overburden at each pier. Fifteen of the piers used single circular cofferdams,

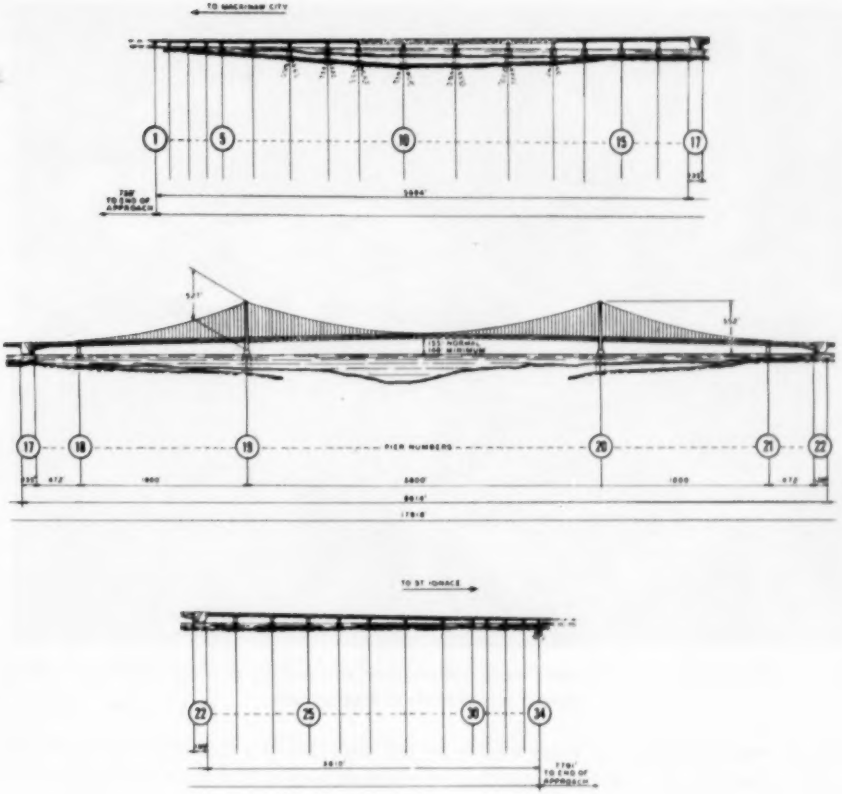

Fig. 1—Mackinac bridge elevation

15 used single rectangular cofferdams, and the northern cable-support pier required a double circular cofferdam.

The internal steel frameworks required to resist the external pressures of the overburden during excavation and the internal pressures during concreting operations, were prefabricated on shore and barged to the site. These sections were then lowered into exact place by floating derricks. With the framework in position (Fig. 2) the surrounding steel sheet piling was driven to bedrock, after which the overburden was removed. If bearing piles were required, the sheet piling was driven 20 ft below mudline, the overburden was removed to 10 ft below mudline, and the bearing piles were driven to refusal in a radial pattern within the cofferdam.

Caisson construction

The steel caissons were of the open-dredge type; that is, they were pierced vertically from top to bottom by open wells, through which the overburden could be excavated by digging-buckets operated by floating derricks. A plated bottom around the wells provided buoyancy. The lower section of the

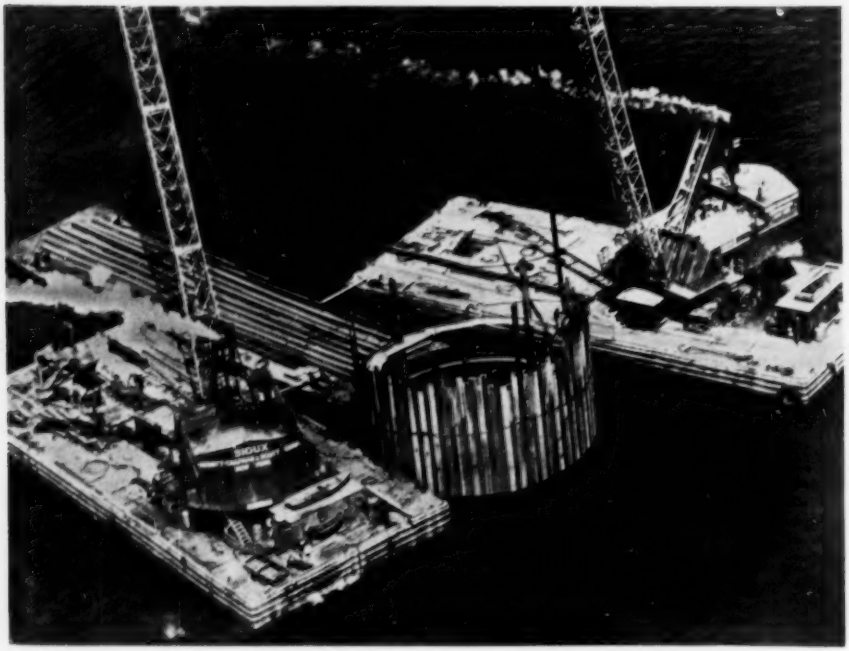

Fig. 2—All but three of the piers were cofferdams, formed by driving steel sheet piling around internal steel framework

caisson was the cutting edge which sliced into the overburden like a chisel as the caisson was sunk into position. The overburden was then removed from beneath the caisson by dredging through the wells, and additional concrete was added to the buoyancy chambers, causing the caisson to sink deeper into the overburden. Exterior walls and digging-wells of the caisson were progressively built up so that their tops were always above water.

The caisson for each of the tower piers was in the form of a ring with an outside diameter of 116 ft and an inside diameter of 86 ft. The 15-ft annular space formed by the two concentric shells was reduced to a thin cutting edge 116 ft in diameter (Fig. 3) and the annular space was divided into eight watertight compartments by radial diaphragms. The 86-ft diameter space in the center served as one huge dredging well.

Prior to positioning the initial section of prefabricated caisson, there was constructed a "corral" system of three towers, fashioned out of pipe and steel H-piles driven vertically to refusal around the perimeter of the area to be occupied by the caisson. After the caisson was floated into position a fourth tower was constructed to close the "corral."

Once exactly above the pier site, the caisson was "bottomed" by pumping water into the compartments, which had been previously filled with prepacked concrete to a height of 22½ ft above the cutting edge. When the top of the

caisson neared water level, the sinking operation was stopped, and the ring of the caisson was built up by adding prefabricated sections (Fig. 4). This process was repeated until the cutting edge was firmly embedded in the overburden. By progressively dredging, placing concrete in the compartments, and building up the walls, the caisson was sunk through the overburden to bedrock. The depth of overburden ranged from approximately 100 ft for the northern tower to 50 ft for the southern tower.

The caisson for the cable support pier on the southern side of the main channel (Fig. 5) was rectangular in shape, 92 ft long by 44 ft wide, and had 21 dredging wells 9 ft in diameter, in three rows of seven each. It was sunk in substantially the same manner as were those for the two tower piers. It was founded at a depth of 145 ft below the surface.

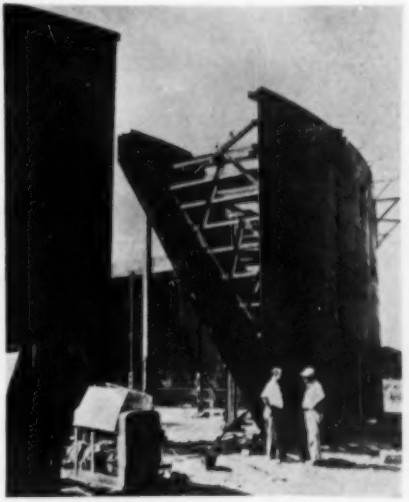

Fig. 3—Cutting edges of main tower caissons were fabricated on land and excavated to floatation at the lake shore at Alpena, Mich., then towed almost 100 miles to the bridge site

Concreting operations

After the caissons or cofferdams were cleaned out to bedrock, or to 10 ft below the mudline where bearing piles were used, a system of vertical grout

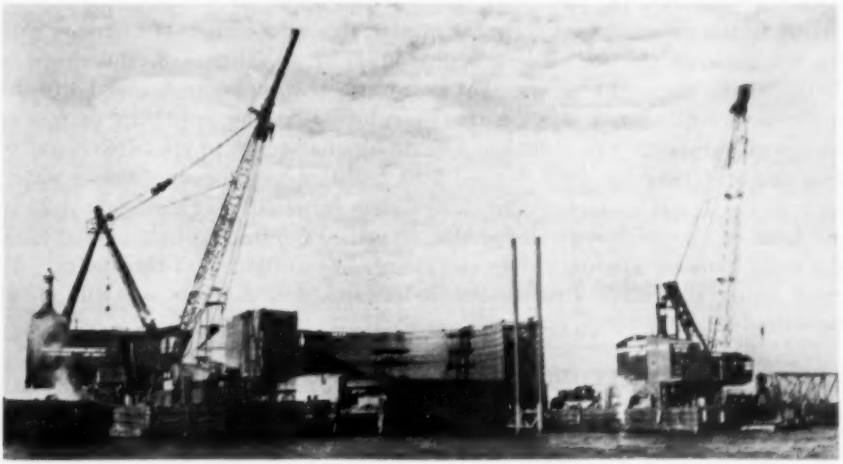

Fig. 4—Main tower caissons, except for cutting edges, were fabricated in position in a corral of pipe and steel H-piles driven to refusal around the perimeter

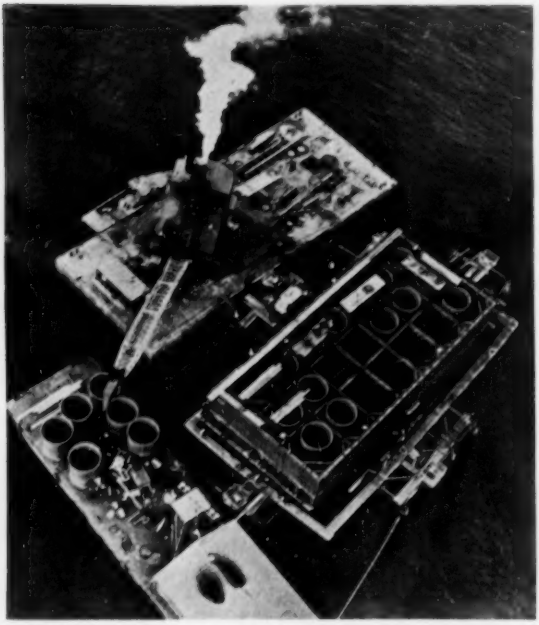

Fig. 5—Because of depth of water, rectangular caisson with 21 dredging wells was used for southern cable support pier

pipes was installed, and coarse aggregate was placed to a predetermined height. The aggregate was then consolidated by grout mixed in, and pumped from, a floating plant. Up to elevation − 13 the concrete was "Class B" for which the required compressive strength at 90 days was 3000 psi. Above elevation − 13, the concrete was "Class A" for which the required compressive strength was 4000 psi.

Due to the severe ice action in the straits, the cross section of all piers, with the exception of the anchorages, was reduced at elevation −8, and single or double shafts encased with wrought-iron protection plates were erected to the +10 elevation. From this point, the concrete in the approach piers was continued either as two cylindrical shafts connected by a spandrel beam or as a single rectangular shaft topped by a hammer-head beam. Above water, each of the 116-ft diameter main tower piers terminates at elevation +25 in the form of two 38-ft diameter shafts. Vertical reinforcing bars extend from the main block at approximately elevation −30 to the top of the shafts. All sheet piling and other steel in the cofferdams and caissons was cut off at elevation −8.

CONCRETING MATERIALS AND GROUT MIXES

Prior to concreting operations, an extensive investigation was made to locate suitable aggregates and to determine (1) the properties of fresh grouts, (2) the void content of the coarse aggregate, and (3) the compressive strength of prepacked concrete as affected by various grout mixtures.

Coarse aggregate

The coarse aggregate selected was a crushed dolomite obtained from a plant approximately 60 miles from the bridge site. It was processed in two sizes, ¾ to 1 ⅞ in. and 2 to 5½ in. These sizes were recombined in approximate proportions of 25 percent of the smaller to 75 percent of the larger by belt blending as the aggregate was loaded on board ship at the aggregate plant. The grading of the aggregate as furnished and as required by specifications is shown in Table 1.

TABLE 1—SIEVE ANALYSIS OF COARSE AGGREGATE

Sieve size, in.	Cumulative percent passing	
	Average at plant	Specification requirement
6	100	95–100
3	67	60–75
1½	40	30–45
¾	6	0–10
½	2	0–1

All coarse aggregate was transported from the processing plant to the job site by self-unloading ships with capacities of from 10,000 to 12,000 tons, which deposited the material either directly into the forms or in a stockpile on a shoal in the lake. Because clean coarse aggregate free from excessive amount of undersized material was required and because the dolomite tended to break with handling, it was necessary to wash the material over a ¾-in. grizzly screen as it left the ship. In addition, all stockpiled material used in Class A above-water concrete, where rehandling was necessary, was washed through a ¾-in. trommel screen. The trommel was used in preference to a flat vibratory screen because of its superiority in producing a well-blended product.

Sand

The sand was also a manufactured product from the same source as the coarse aggregate. It was transported in separate compartments of the same self-unloading ships and was discharged either onto barges or into an old ore-boat hull that was purchased for the job.

During the course of the work the grading of the sand and its fineness modulus were subject to undesirably large variations as shown by the analyses of Table 2. In particular, at times the percentage of −100 mesh material was on the high side and led to larger than normal water requirement of grouts of given cement content. In September, 1955, the sand grading was changed

TABLE 2—SIEVE ANALYSES OF GROUT SANDS

Sieve No.	Average cumulative percent passing			
	Aug.-Nov. 1954	Apr.-Aug. 1955	Sept.-Nov. 1955	Specification requirement
12	98.6	97.6	96.4	100
14	97.8	96.1	94.1	90–100
30	65.5	55.9	47.7	55–75
50	38.7	36.0	26.7	25–45
100	17.7	20.2	13.5	5–20
200	5.7	6.4	5.0	0–5
FM	1.91	1.94	2.15	1.65–2.10

so as to reduce the −100 mesh material. This resulted in a substantial reduction in water requirement of grout mixes and increase in compressive strength of prepacked concrete, of which more will be said later.

Cementing materials

The portland cement was Type I meeting the requirements of ASTM C 150.

The fly ash obtained from the Detroit area was a remarkably uniform product, at all times meeting the chemical and physical requirements of the specifications. The principal requirements are shown in Table 3.

Among 91 samples representing about 13,000 tons, the loss on ignition varied from 1.8 to 3.8, with an average of 2.8 percent, and the specific surface varied from 2840 to 4160, with an average of 3120 sq cm per g.

TABLE 3—FLY ASH SPECIFICATION REQUIREMENTS

Silica, percent, minimum	40.0
Alumina, percent, minimum	15.0
Magnesia, percent, maximum	3.0
Sulfur trioxide, percent, maximum	3.0
Loss on ignition, percent, maximum	4.5
Moisture content, percent, maximum	2.0
Specific gravity, percent, minimum	2.3
Specific surface, sq cm per g, minimum	2800
Water requirement, percent increase, maximum	3.0

Grout mixes

At the beginning of the work the grout mix for Class B concrete (below elevation −13) was composed of 2½ parts of cement to 1 of fly ash to 4.2 of sand on a weight basis. As the results of strength tests of prepacked concrete cylinders became available, the quantity of cementing material was reduced, the leanest mix being composed of 1.6 cement to 1 fly ash to 4.2 sand on a weight basis. Utilizing this latter mix, the portland cement content of Class B underwater concrete was about 2.4 sacks per cu yd, and cement plus fly ash content was the equivalent of about 4.3 sacks per cu yd.

For Class A concrete (above elevation −13) the grout mix generally used was composed of 2½ parts of cement to 1 part of fly ash to 4.2 parts of sand measured by weight, though during the colder months a 3.8 to 1 to 5.5 mix was used.

Data concerning the various grout mixes used in the work are shown in Table 4.

TABLE 4—COMPOSITION OF CONCRETE

	Grout mix, proportions by weight of cement: fly ash: sand				
	3.8:1:5.5	2.5:1:4.2	2.2:1:4.2	1.9:1:4.2	1.6:1:4.2
Water/(cement + fly ash)	0.54	0.56	0.57	0.60	0.62
Cement factor, sacks per cu yd*	4.5	3.9	3.5	2.7	2.4
Fly ash factor, sacks per cu yd*	1.5	1.9	2.1	1.8	1.9
Sand factor, cu ft per cu yd*	6	5.8	6.2	5.4	5.7

*Data based on 37.5 percent void content of coarse aggregate in Class B and 40 percent void content in Class A concrete.

AGGREGATE PLACEMENT

Prior to the placement of the coarse aggregate, 1¼- or 2-in. vertical grout pipes on 10- to 20-ft centers in each direction were fixed in position in cofferdams and caissons. Also, vertical slotted pipes or "sounding wells" were installed at desired locations. These sounding wells made it possible to determine the level of the grout within the aggregate mass at any time and hence made it possible to control the rate of grouting at any location as desired.

With the grout pipes and sounding wells in position, the coarse aggregate was placed (Fig. 6). For the underwater concrete operations on the anchorages, cable-support piers, main tower piers, and some of the larger approach piers, the material was deposited directly from a self-unloading ore ship. The ship was anchored beside a pier; and by the simultaneous manipulation of the vessel with deck winches and its 36-in. swivel-boomed belt conveyor, the aggregate was placed in approximately horizontal lifts. The ability of these self-unloading ore ships to rapidly place a large volume of coarse aggregate was a tremendous asset to the job, enabling large savings in placement costs. Also, the rapid filling with aggregate stabilized the cofferdams and thus prevented possible storm damage. By this method, approximately 10,000 tons of aggregate could be deposited in a pier in 6 hr.

Coarse aggregate was placed in the smaller approach piers and for the bulk of the above-water concrete (where it was not practical to deposit the material directly from ore ship) by clam buckets operated by floating derricks.

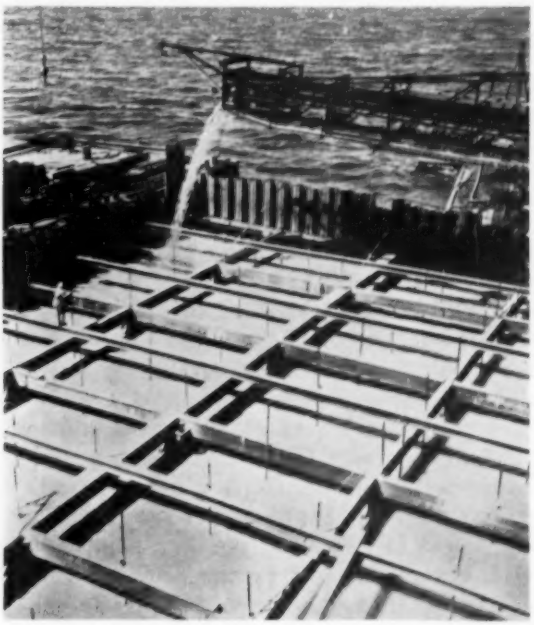

Fig. 6—Coarse aggregate falls into massive rectangular cable anchorage pier from conveyor of self-unloading ship. Sheet-pile bracing frame doubled as access to intrusion insert pipes

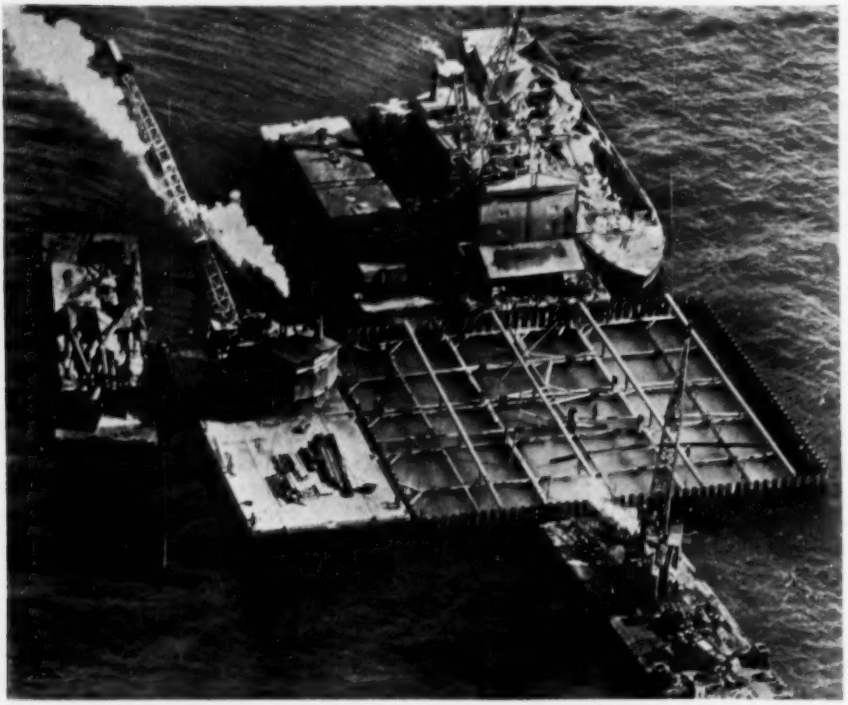

Fig. 7—Sand and covered cement–fly ash barges attend the mixing and pumping plant as it prepares and pumps grout at one of the two cable anchorage piers

The material was clammed from a stockpile, passed through a trommel wash-screen on to a barge which was towed to the site of operations.

GROUT PLANT

One of the primary considerations in favor of the prepacked method was that approximately 60 percent of the concrete materials, as represented by the coarse aggregate, could be placed directly in the forms from barge or ship. The remaining 40 percent could be handled, batched, mixed, and placed with a much smaller plant for the same over-all rate of concrete production than would be required for conventional concrete. The necessity for rapidly consolidating relatively large masses of aggregate sometimes under difficult weather conditions required a grouting plant of unprecedented size, peak production reaching 260 cu yd of grouted aggregate per hour.

The grout plant (Fig. 7) was mounted on a derrick barge with the bins, batchers, mixers, agitators, and pumps located on the forward half, the derrick amidship, and the generators, compressors, and bulk cement-handling equipment astern. The sand was conveyed from a barge alongside by a clam bucket operated by the derrick. The bulk cement and fly ash in a covered partitioned barge on the opposite side were pumped to the bins.

The dual plant consisted of two semi-automatic weight batchers, two 3-cu yd mixers, two 4-cu yd agitators, and six positive displacement grout pumps and was equipped to produce steam and hot water for winter operations.

GROUT MIXING

Batched materials were added to the mixer in the following order: (1) water and intrusion aid, (2) cement and fly ash, and (3) sand. The grout was mixed for 2½ min and discharged to agitator. The consistency of the grout was controlled by flow-cone test, the result of which is expressed as the time in seconds for a given quantity of grout to flow through a ½-in. diameter orifice at the apex of an inverted cone. This test with its "flow factor," in seconds, for intrusion grouts corresponds to the slump test, with slump in inches, for conventional concretes. For this work it was specified that the flow factor be not less than 17 nor more than 24 sec. On the average the flow factor was about 21 sec.

Also, moisture content of sand as delivered to mixer was determined frequently to properly control batch weights.

GROUTING

After the coarse aggregate was placed in a lift of predetermined height in a cofferdam or caisson, the floating grout plant was moored alongside and the grout was pumped through flexible hose lines (Fig. 8) to the vertical grout-insert pipes. For each lift, grouting of the unconsolidated aggregate

Fig. 8—Grout is conveyed from floating plant to pier through 2-in. hoses and pipes

proceeded from the lowest point in the unconsolidated mass to a point just below the top of the aggregate, where it was brought approximately to a uniform elevation (as judged by soundings in inspection pipes). The grout surface could be accurately located by a sounding line and float which would sink in water but float on grout. The vertical grout pipes were raised as the grout surface advanced so that the point of discharge was 2 to 5 ft below the grout surface. At the completion of a lift, the intrusion grout pipes were raised above the grout level and washed out.

Placing of coarse aggregate and consolidation by grouting were often carried on simultaneously. When it became necessary to interrupt the grouting of a lift due to severe weather conditions or for placements scheduled on other piers, the grout pipes were raised and washed clean. Later, when the operation was resumed, the grout pipes were worked down to the top of the mass previously grouted. There was no concern over cold joints as low compressive stresses were involved and, at the joint, pieces of coarse aggregate which were only partially embedded in the lift below became completely embedded as the lift above was grouted, thus assuring good shearing resistance.

To take full advantage of the short construction season, all operations were executed on a 7-day week, three-shift basis until August, 1955. By that time, all underwater concrete was in place and, with only relatively small placements involved, concreting operations were reduced first to two shifts and then to one shift a day.

CONCRETE PLACEMENT RECORDS

The advantages of the prepacked method in rapidly placing large volumes of concrete under water of great depth became increasingly apparent as the work progressed. The compact grout plant with its flexible grout lines was able to continue operations under adverse weather conditions which would have made it impossible or extremely hazardous to operate a conventional concrete plant. On several occasions, two of the smaller approach-pier foundations were pumped simultaneously.

During the spring of 1955, when the main tower piers were being constructed, an all-time record for placement of underwater concrete from a floating plant was established when 6250 cu yd of aggregate was consolidated in 24 hr at an average rate of 260 cu yd per hr. In 30 working days, 102,000 cu yd of concrete was produced.

Above-water concrete

Production of concrete in the formed sections above water was the same as for the below-water work, except that the insert pipes were placed on 5- to 10-ft centers, so that the grout could the better be brought to a horizontal surface just below the top of each lift of forms. After the forms had been raised and the reinforcing steel had been placed for the next lift, the intrusion pipes were reset. At finished surfaces, a surplus of mortar was brought to

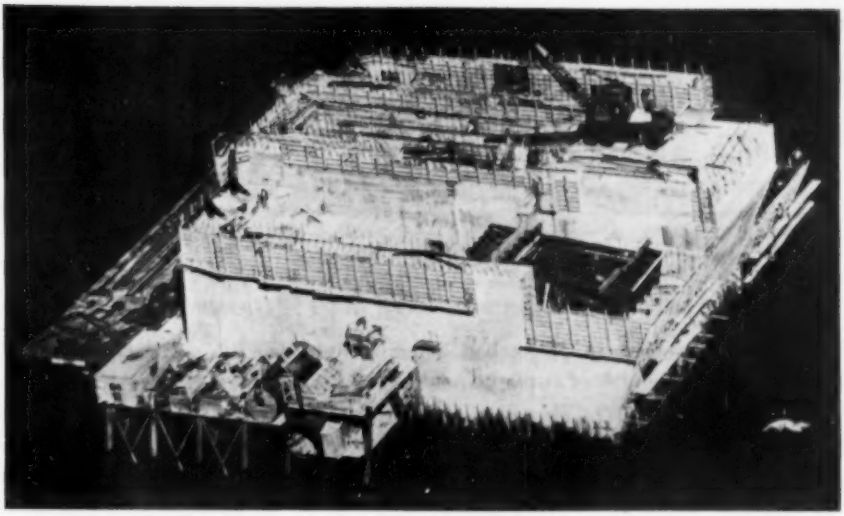

Fig. 9—Suspension cables will be divided and secured to the cable anchor bars shown partially embedded in the massive cable anchorage pier

the top of the coarse aggregate, and sufficient pea gravel was worked into the grout to produce a topping mix that was screeded and floated to finished grade. Forms were vibrated with a high-frequency vibrator.

Reusable steel forms were employed for the major portion of the job; wood forms were used for detail work or where repetition of operation was insufficient to warrant the use of steel forms (Fig. 9). All forms were designed for a full liquid-grout head. Special attention was paid to the tightness of forms since the intrusion grout was as capable of penetrating small holes as of filling voids in the aggregate.

COMPRESSIVE STRENGTH

As an index to the quality of prepacked concrete in the work, the compressive strengths of 6 x 12-in. job cylinders were determined at ages of 7, 28, and 90 days. These specimens were prepared by first filling steel molds with aggregate ranging in size from ½ to 1½ in. and then consolidating the aggregate with grout sampled from the mixing plant. The average compressive strengths of job cylinders of prepacked concrete for the several grout mixes employed are given in Table 5.

From a study of the table it will be noted that the gain in strength between 28 and 90 days is much greater than is normally the case for conventional concretes employing Type I cements and the larger the proportion of fly ash the greater the percent gain in strength. Also, it is evident that the ratio of weight of water to that of cementing materials (cement plus fly ash) in-

TABLE 5—COMPRESSIVE STRENGTH OF PREPACKED CONCRETE

	Grout mix proportions by weight (cement: fly ash: sand)				
	3.8:1:5.5	2.5:1:4.2	2.2:1:4.2	1.9:1:4.2	1.6:1:4.2
$W/(C+F)$:	0.54	0.56	0.57	0.60	0.62
Age, days	Compressive strength, psi				
7	2290	1920	1720	1590	1360
28	3480	3220	2980	2830	2250
90	4470	4310	4270	3660	3470

fluences the strength of prepacked concrete much as the water-cement ratio influences the strength of conventional concrete.

The values of $W/(C+F)$ were greater than are normally required for comparable grout mixtures containing well-graded natural sands, and it appears that the shape of sand particles may have a rather large influence upon the water requirement. Also, changes in grading of manufactured sand, particularly in the percentage passing the No. 100 sieve, markedly affect the water requirement of intrusion grouts. This is illustrated by the job experience of the summer of 1955. During July, the percentage of −100 material in the sand averaged slightly more than 20. A marked increase in water demand was observed, and subsequently the sand grading was changed so that only about 13 percent passed the No. 100 sieve.

The effect upon water requirement of 2½ to 1 to 4.2 grout and compressive strength of prepacked concrete is shown in Table 6.

TABLE 6—RELATION OF W/(C+F) TO STRENGTH

Month	$W/(C+F)$ by weight	Compressive strength, psi		
		7-day	28-day	90-day
July	0.59	1420	2350	3720
September	0.54	2230	3690	4630

In addition to the compressive-strength tests on 6 x 12-in. cylinders, several cores, 10 in. in diameter by 20 ft long, were taken from the anchorages. Inspection showed good consolidation of the coarse aggregate and satisfactory bond between construction lifts. The compressive strength of 10 x 20-in. samples taken from the larger cores varied from 2100 to 3800 psi. These strengths were entirely satisfactory, considering the size effect of the 5½-in. maximum aggregate.

CONCLUDING STATEMENTS

Under the conditions encountered on the Mackinac Bridge project, which, during a short construction season, necessitated the placement of 440,000 cu

yd of concrete in 34 piers, in water of depths as great as 200 ft, the prepacked method possessed advantages as follows:

1. Since it was necessary to run only 40 percent of the materials through the mixers, the plant cost was substantially less than it would have been for a conventional plant of comparable production output.
2. The cement factor was reduced from a probable 7 sacks per cu yd mix for conventional tremie concrete to as low as 2.4 sacks per cu yd for the underwater prepacked mix of which fly ash was a constituent.
3. Rapid placement of the coarse aggregate made it possible to quickly stabilize cofferdams and caissons and thus to protect them from potential storm damage.
4. The possibility of pumping through flexible grout lines often allowed operations to continue when it would have been impossible or extremely hazardous to place conventional concrete.
5. Operations could be easily and quickly interrupted by raising the vertical insert pipes above the grout level.

ORGANIZATION

The project was constructed by the Mackinac Bridge Authority whose chairman was Prentiss M. Brown and secretary was Lawrence A. Rubin.

J. W. Kinney was the resident engineer for D. B. Steinman, consulting engineer, New York, whose contract included field engineering and inspection as well as design.

The substructure contract of the Merritt-Chapman & Scott Corp. was under the direction of William Denny, executive vice-president; Grover C. Denny, project manager; and C. E. Haltenhoff, project engineer.

The Prepakt Concrete Co., Cleveland, working with Merritt-Chapman and Scott Corp. on the concrete placement was represented by Paul D. Smith, executive vice-president; Mered McKinzie, superintendent; and R. E. Davis, Jr., project engineer.

For such discussion of this paper as may develop please see June 1957 JOURNAL.

Title No. 53-32

Ultimate Flexural Analysis Based on Stress-Strain Curves of Cylinders*

By G. M. SMITH and L. E. YOUNG†

SYNOPSIS

Describes a stress block for ultimate load analysis based upon the stress-strain relation of 6 x 12-in. cylinders. The stress-strain relation from a cylinder, which includes a decrease in stress beyond the ultimate, is described by a single continuous function. The function is used to compute the total compressive force in the compression zone, position of neutral axis, and ultimate moment. The computed values of total compressive force, neutral axis location, and ultimate moment are compared with actual test values of reinforced concrete beams. The mode of failure is predicted for beams of near balanced design.

The approach described leads to a more accurate analysis of beams reinforced with more than one layer of steel.

INTRODUCTION

The preponderance of knowledge regarding ultimate strength of reinforced concrete members is based on test results, and any theory of ultimate strength is limited in its applicability by the scope of those tests. Ultimate theories proposed have evolved from an assumed stress distribution in the compression zone (stress block) as described by mathematical functions or empirical parameters. Ideally one should be able to start with the properties of the concrete and steel as obtained from simple tests, and through a mathematical analysis compatible with the physical boundaries, accurately predict the ultimate loads that a structural member will withstand. This procedure should be valid for the large variations in concrete properties resulting from the use of different cements, aggregates, mix proportions, and curing conditions and a variety of steel properties and reinforcement locations.

This paper deals only with the flexural strength of rectangular reinforced concrete beams. It is an attempt to predict the ultimate loads that such beams can support based upon a mathematical analysis related to the properties of the materials.

Over the years a large number of shapes of the stress block have been advanced. To support each of these theories, limited experimentation has

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†Members American Concrete Institute, Assistant Professors, Engineering Mechanics Department, University of Nebraska, Lincoln, Neb.

been used and in most cases empirical expressions derived that give results agreeing with test values. This procedure limits the confidence that can be placed in the empirical results to materials and specimen sizes corresponding to the conducted experiments. The recent work of Hognestad, Hanson, and McHenry[1] indicates that the stress-strain curves of concrete in compressive flexure and in direct compression show a striking similarity for a wide range of concrete strengths.

It is more than pure speculation then to assume that the shape of the stress block in flexure corresponds to the stress-strain curve that might be obtained from a cylinder using certain testing techniques. That portion of the stress-strain data of a cylinder beyond the ultimate strength is limited in its availability by the lack of stiffness of the testing machine and the limitations of time in which to obtain data. The sudden failures observed in compression tests of cylinders have been attributed to the release of energy stored in the testing machine.[2] However, in flexural tests the loads are relatively small in comparison to loads in compression tests and little energy is released from the testing machine in flexural tests. By using suitably stiff testing machines it is believed that stress-strain relations beyond the maximum load can be obtained that closely parallel the stress block in flexure.

ANALYTICAL ANALYSIS

Notation

A_s = total area of tension reinforcement; A_t and A_b area of top and bottom layers of steel, respectively

b = width of rectangular member

d = distance from centroid of steel to compression edge of member; d_t and d_b distance to top and bottom layers of steel, respectively

f_{sp} = yield stress of steel

E_s = modulus of elasticity of steel (30×10^6 psi)

ϵ_s = strain in tension steel within the elastic limit; ϵ_t and ϵ_b strain in top and bottom layers of steel, respectively

ϵ_{sp} = strain in tension steel at beginning of yield point

T = total tension force in the steel; T_t and T_b tension force in top and bottom layers of steel, respectively

f_c' = compressive strength of 6 x 12-in. concrete cylinder

ϵ_o = concrete strain corresponding to f_c' as determined from test of cylinder

ϵ_u = ultimate concrete strain in flexure

ϵ = variable strain in the compression block

f = variable stress in stress block

e = natural logarithm base = 2.718

k = ratio of distance to neutral axis to depth of beam

k_b = ratio of distance to neutral axis to depth of beam for balanced design

C = total compressive force of stress block

f_a = average compressive stress in stress block

y = variable distance from neutral axis to a compression fiber

y_o = centroid of stress block with respect to neutral axis

M_u = ultimate moment

p = ratio of A_s/bd

β = compression index

$$= \frac{A_s E_s \epsilon_u}{bdf_c'} = \frac{p E_s \epsilon_u}{f_c'}$$

q = tension reinforcement index

$$= \frac{A_s f_{sp}}{bdf_c'} = \frac{p f_{sp}}{f_c'}$$

Analysis

It was found that the stress-strain relation

$$f = \epsilon M e^{-N\epsilon}$$

closely paralleled the results of test cylinders which included a decrease in stress beyond the ultimate. M and N are constants evaluated in terms of the concrete properties by the following boundary conditions:

$$f = f_c' \text{ when } \epsilon = \epsilon_o$$

$$\frac{df}{d\epsilon} = 0 \text{ when } \epsilon = \epsilon_o$$

From the above boundary conditions the function then becomes

$$f = f_c' \frac{\epsilon}{\epsilon_o} e^{1-\epsilon/\epsilon_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

In Fig. 1 are shown the stress-strain curves of four concrete strengths

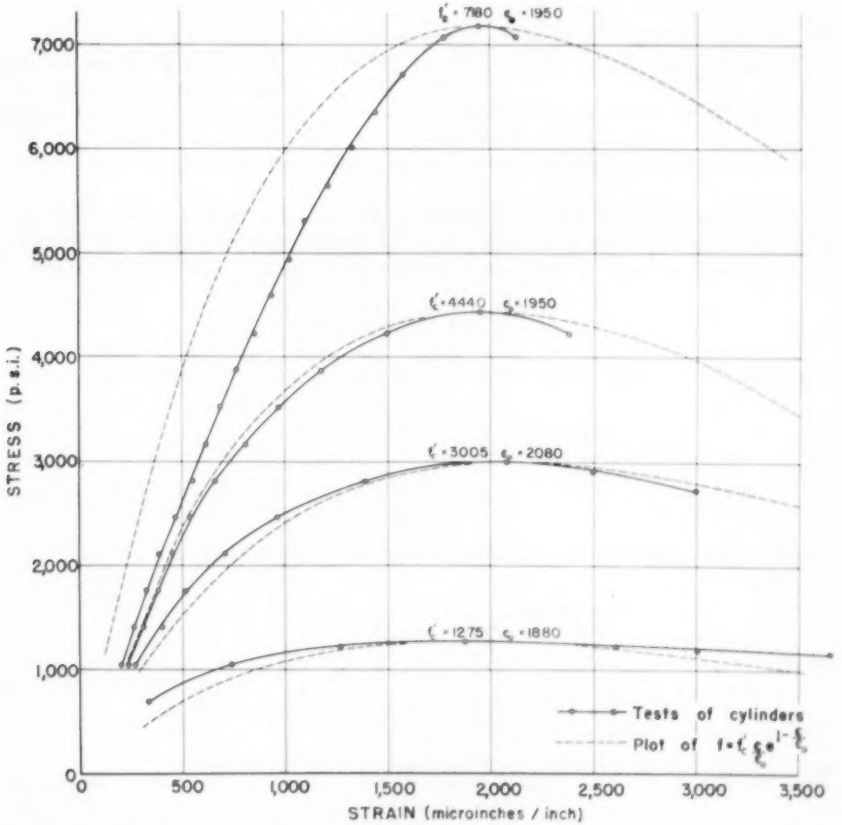

Fig. 1—Comparison of stress-strain curves of cylinders with those plotted from Eq. (1)

and the corresponding plot of the function shown in Eq. (1). It is to be noted that for the weaker concretes, an extension of the cylinder test curve far beyond ultimate strength was obtained but for a very strong concrete this extension was limited. The cylinder tests were conducted at a slow loading rate of about 5000 lb per min in a 440,000-lb testing machine. The plot of the function corresponds well to the cylinder tests except for the case of the very strong concrete where the difference in areas under the curve is about 10 percent. The stress block will then correspond to this function to a point where the ultimate flexural strain, ϵ_u occurs.

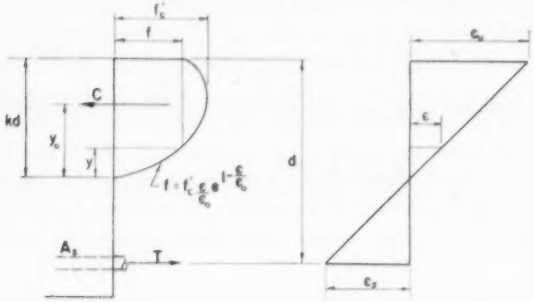

Fig. 2—Distribution of stress and strain on a section of a reinforced beam at ultimate load

Proceeding with the mathematical derivations based on the stress and strain distributions on a section of a reinforced concrete beam at ultimate as illustrated in Fig. 2, the total compressive force C can be expressed as:

$$C = b \int_0^{kd} f \, dy$$

where

$$\frac{y}{\epsilon} = \frac{kd}{\epsilon_u} \text{ and } dy = \frac{kd}{\epsilon_u} d\epsilon$$

Then

$$C = \frac{b\,k\,d}{\epsilon_u} \int_0^{\epsilon_u} f \, d\epsilon$$

Substituting f from Eq. (1), integrating between limits, and simplifying:

$$C = b\,k\,d\,f_c' \, e \frac{\epsilon_o}{\epsilon_u} \left[e^{-\epsilon_u/\epsilon_o} \left(-\frac{\epsilon_u}{\epsilon_o} - 1 \right) + 1 \right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

where

$$f_c' e \frac{\epsilon_o}{\epsilon_u} \left[e^{-\epsilon_u/\epsilon_o} \left(-\frac{\epsilon_u}{\epsilon_o} - 1 \right) + 1 \right] = \text{average stress, } f_a$$

and the total compressive force may be written as follows:

$$C = bkdf_a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

In Fig. 3, f_a/f_c' is evaluated for various values of ϵ_o/ϵ_u.

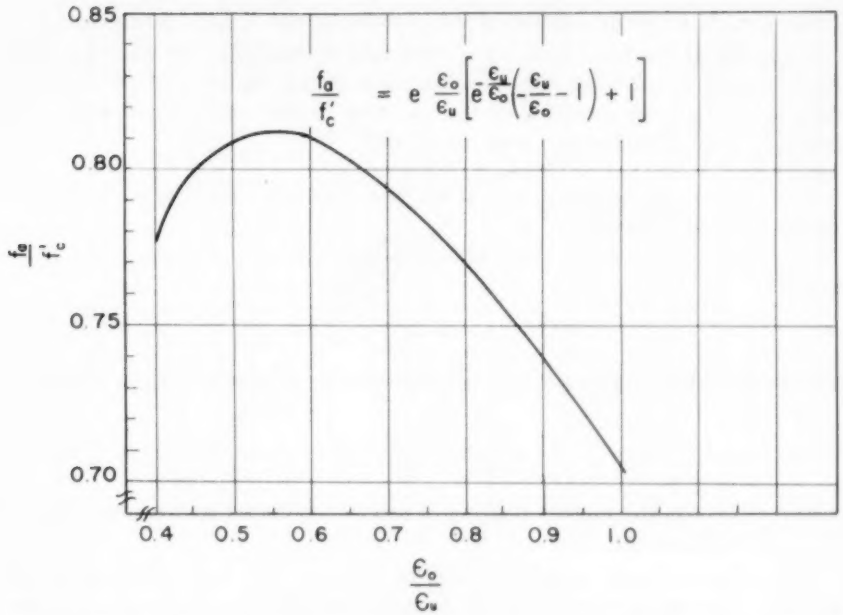

Fig. 3—Ratio of average stress in flexure to ultimate compressive stress from a cylinder

To locate the neutral axis for a concrete failure the strain in the top fibers is taken to be ϵ_u and by similar triangles the strain in the steel becomes:

$$\epsilon_s = \left(\frac{1-k}{k}\right)\epsilon_u$$

From equilibrium the total tensile force in the steel equals the total compressive force in the stress block and:

$$C = T = A_s E_s \epsilon_s = A_s E_s \left(\frac{1-k}{k}\right)\epsilon_u \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Equating Eq. (3) and (4)

$$bkdf_a = A_s E_s \left(\frac{1-k}{k}\right)\epsilon_u$$

and letting:

$$p = \frac{A_s}{bd}$$

and simplifying:

$$\frac{f_a}{f_c'}k^2 + p\,\frac{E_s \epsilon_u}{f_c'}k - p\,\frac{E_s \epsilon_u}{f_c'} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

$(pE_s\epsilon_u)/f_c'$ is the familiar compression index β, and Eq. (5) then becomes:

$$\frac{f_a}{f_c'}k^2 + \beta k - \beta = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

From Fig. 4, k can be evaluated for various values of f_a/f_c' and β.

To locate the neutral axis for a steel failure, consider the ultimate strain conditions to occur when the steel has yielded and the neutral axis has risen sufficiently to cause crushing of the concrete in the compression zone. The strain in the top fibers will then be ϵ_u and

$$T = A_s f_{yp}$$

Again from equilibrium:

$$bkdf_a = A_s f_{yp}$$

and if

$$p = \frac{A_s}{bd}$$

and multiplying numerator and denominator by f_c' the following results:

$$k = \frac{p f_{yp} f_c'}{f_c' f_a} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

where pf_{yp}/f_c' is the tension index q; k can be evaluated for various values of f_a/f_c' and q from Fig. 5.

It is often difficult to predict whether a beam will be a concrete or steel failure. The strain conditions for simultaneous failure of concrete and steel

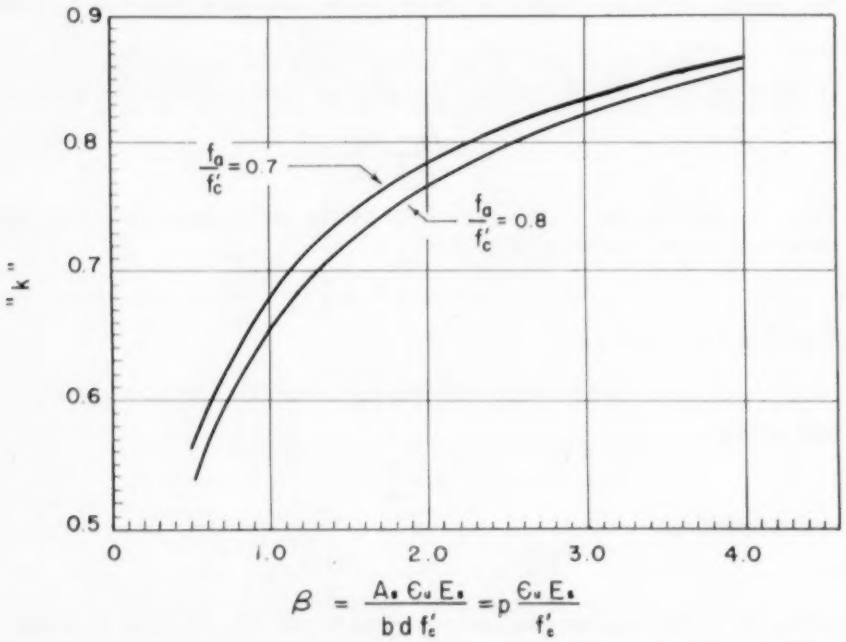

Fig. 4—Factor k, ratio of distance to neutral axis to depth of beam, as per compression index

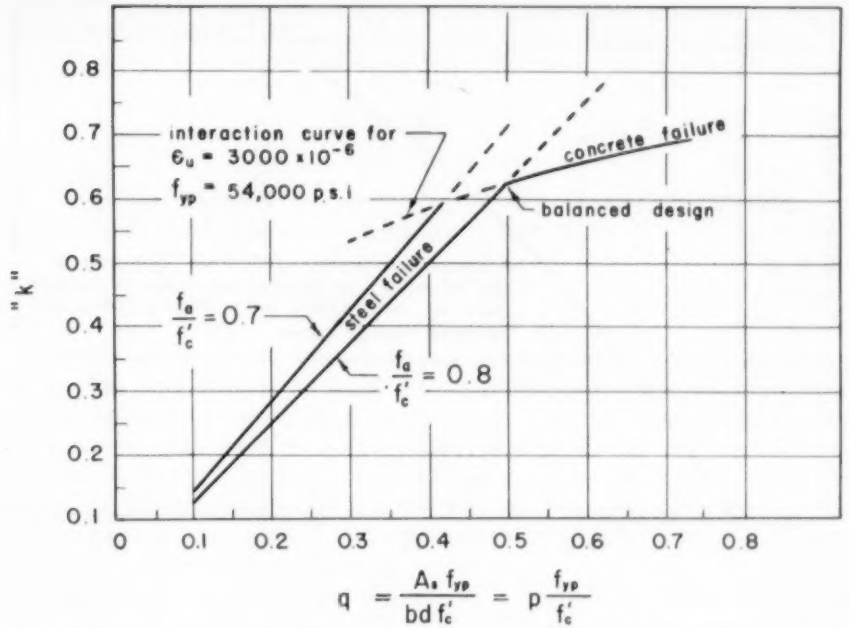

Fig. 5—Factor *k*, ratio of distance to neutral axis to depth of beam, as per tension reinforcement index

occur when the strain in concrete becomes ϵ_u and the strain in the steel becomes ϵ_{yp}. Then by similar triangles

$$\frac{\epsilon_u}{\epsilon_{yp}} = \frac{k_b}{1 - k_b} \qquad (8)$$

Fig. 6 shows various values of k_b for balanced design corresponding to various values of ϵ_u/ϵ_{yp}. An under-reinforced section (steel failure) will have a value of k, as determined from either the compression index or steel index, less than k_b for balanced design. Conversely, an over-reinforced beam will have a k value greater than k_b.

The centroid of the stress block or the location of the resultant compressive force is determined as follows:

$$y_a = \frac{b \int_0^{kd} y\, f dy}{C} = \frac{b \left(\frac{kd}{\epsilon_u}\right)^2 \frac{f'_c}{\epsilon_o} \int_0^{\epsilon_u} \epsilon^2 e^{1-\epsilon/\epsilon_o} d\epsilon}{C}$$

Integrating and simplifying:

$$\frac{y_a}{kd} = \frac{e^{-\epsilon_u/\epsilon_o}\left[-1 - 2\frac{\epsilon_o}{\epsilon_u} - 2\left(\frac{\epsilon_o}{\epsilon_u}\right)^2\right] + 2\left(\frac{\epsilon_o}{\epsilon_u}\right)^2}{\frac{\epsilon_o}{\epsilon_u}\left[e^{-\epsilon_u/\epsilon_o}\left(-\frac{\epsilon_u}{\epsilon_o} - 1\right) + 1\right]}$$

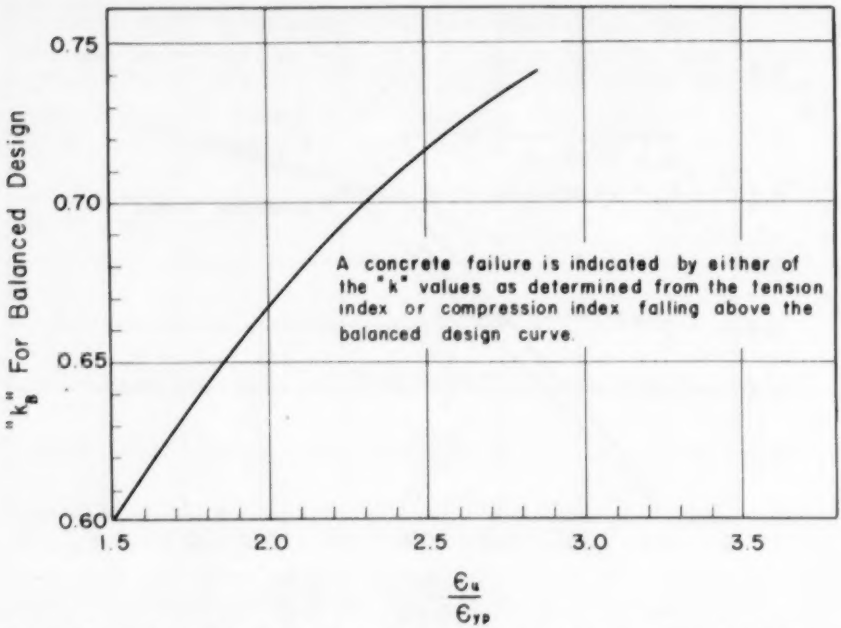

Fig. 6—Factor k_b, ratio of distance to neutral axis to depth of beam, for balanced design

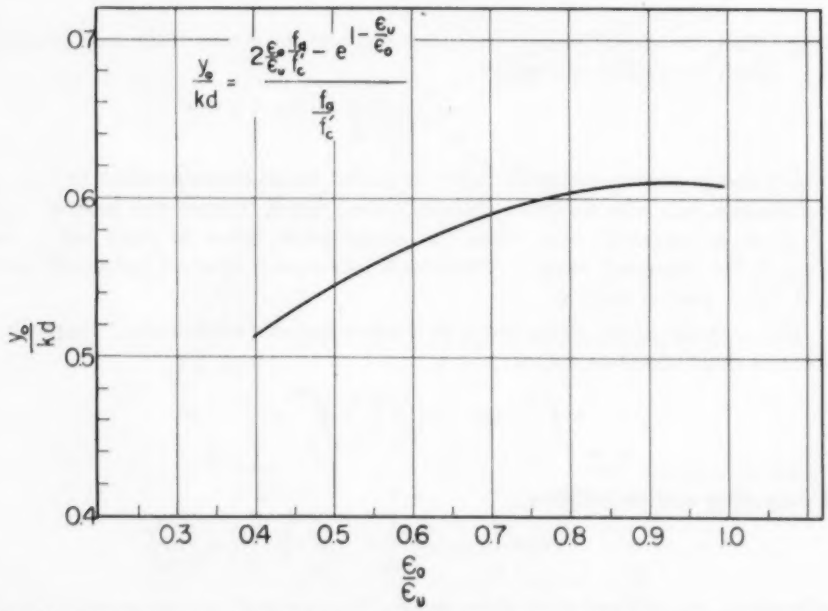

Fig. 7—Ratio of y_o, location of centroid of "stress block," to kd, distance to neutral axis

which may be reduced to:

$$\frac{y_o}{kd} = \frac{2\frac{\epsilon_o}{\epsilon_u}\frac{f_o}{f_c'} - e^{1-\epsilon_u/\epsilon_o}}{\frac{f_o}{f_c'}} \qquad (9)$$

Fig. 7 can be used to evaluate y_o/kd for various values of ϵ_o/ϵ_u.

The computation of the ultimate moment is based on the equating of the external moment to the internal moment which from Fig. 2 can be seen to be $M_u = T\,(d - kd + y_o)$, for a single layer of steel.

For beams containing more than one layer of steel and near balanced design, the bottom layer or layers may begin to yield while the upper layer may be stressed considerably below the yield point. Such a condition existed in beam C (see Table 1). The strain readings during the test indicated yielding of the lower layer at ultimate while the upper layer had a stress well below the yield point of only 19,500 psi. A theoretical calculation of the strains in each layer of steel indicates a stress of 22,900 psi in the top layer and a yield point stress in the lower layer.

When k and ϵ_u are known the forces in each layer of steel can be found from the calculated strains in each layer of steel as determined by the geometry of similar triangles shown in Fig. 8. If we consider Fig. 8, it is apparent that:

$$T_t = A_t E_s \epsilon_t$$

where

$$\frac{\epsilon_t}{d_t - kd} = \frac{\epsilon_u}{kd}$$

and

$$T_b = A_b E_s \epsilon_b$$

where

$$\frac{\epsilon_b}{d_b - kd} = \frac{\epsilon_u}{kd}$$

Then

$$M_u = T_t\,(d_t - kd + y_o) + T_b\,(d_b - kd + y_o)$$

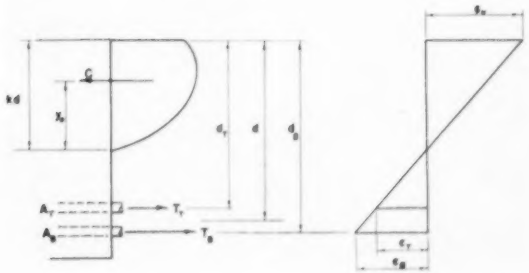

Fig. 8—Stress and strain distribution considering two layers of steel

TABLE 1—EXPERIMENTAL AND CALCULATED DATA FROM BEAM TESTS

Beam	Concrete properties				Steel properties†									
					Strain		Area							
	f_c'	ϵ_u*	ϵ_0	$\frac{\epsilon_0}{\epsilon_u}$	Top layer	Bottom layer	Top layer	Bottom layer	$p = \frac{A}{bd}$	$\beta = \frac{pE_s\epsilon_0}{f_c'}$	$q = \frac{pf_{sy}}{f_c'}$	k from Fig. 4	k from Fig. 5	k_b, balanced design
A	4440	3000	1950	0.65	700	1600	0.758	0.752	0.063	1.28		0.698		0.625
B	4440	3100	1950	0.628	575	1600	0.749	0.742	0.062	1.30		0.70		0.645
C	3050	3050	2000	0.656	650	yield	0.518	0.515	0.043	1.296		0.70		0.642
D	2560	2780	1675	0.602	1505	—	0.764	—	0.032	1.04		0.654		0.62
E	2560	3200	1675	0.524	yield	—	0.726	—	0.030	1.13		0.675		0.653
F	2230	3070	1740	0.567	yield	—	0.524	—	0.022		0.527		0.63	0.643
G	2230	3000	1740	0.580	1500	—	0.527	—	0.022	0.885		0.63		0.638

Beam	Mode of failure		kd		$\frac{f_s}{f_c'}$ Fig. 3	Compressive force and tensile force		y_c from Fig. 7	M_s, inch-kips		
	From Fig. 6	Experimental	Calculated	Experimental		Calc. from stress-strain function	Experimental tensile force from steel strains		Calc.	Exp.	Percent error
A	Conc.	Conc.	4.18	4.10	0.80	57,950	52,000	2.43	264	292	9.6
B	Conc.	Conc.	4.20	4.45	0.80	57,500	48,500	2.42	259.4	275.5	5.9
C	Conc.	Conc.	4.20	4.20	0.80	39,670	37,900	2.35	176.5	208	15.2
D	Conc.	Conc.	3.93	3.90	0.81	32,500	34,500	2.24	145	155	6.4
E	Conc.	Conc. and Steel	4.05	—	0.81	33,600	39,200	2.27	141.6	169.2	16.3
F	Steel	Steel	3.78	—	0.81	27,300	28,300	2.19	120.3	120.8	0.4
G	Conc.	Conc.	3.78	4.00	0.81	27,300	23,700	2.14	118.9	120	0.9

*Obtained from beam tests.
†f_{sy} = 54,000 psi. E_s = 30,000,000 psi.

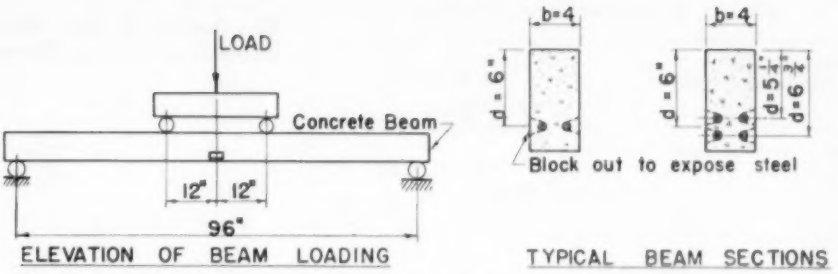

Fig. 9—Loading and dimensions of beams

EXPERIMENTATION

Details of loading and dimensions of the test beams are shown in Fig. 9. Strains in the concrete were measured with 6-in. SR-4 gages on the top and side of the compression zone. One-half-in. SR-4 gages were applied to each steel bar through a blocked out opening (see Fig. 10) in the center section of the beam. The blocked portion around the steel was effective in localizing the failure in the region of the SR-4 gages on the concrete. The deformations on the reinforcement were removed in a lathe to facilitate gage application and area computation. Control cylinders and beams were cast from sand-gravel concrete of various strengths as shown in Table 1. The beams and cylinders were cured 7 days in the moist room and tested dry at about 40 days of age.

Tension tests of the reinforcement steel gave a modulus of 30×10^6 and a yield point of 54,000 psi.

The position of the neutral axis during all stages of loading was determined from the linear strain plots shown in Fig. 11. The instantaneous strain in the top concrete fibers at ultimate was recorded on a brush oscillograph. The strains on the side of the concrete and one steel bar were followed on static analyzers to obtain the instantaneous values at ultimate.

In the testing of cylinders an attempt was made to obtain that portion of the stress-strain curve beyond the ultimate strength. For low strength concretes ($f_c' = 3500$ and under) this is easily available in a relatively stiff

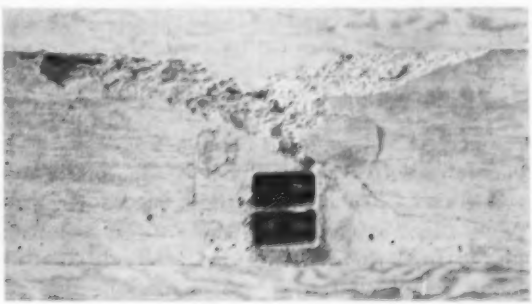

Fig. 10—Typical concrete failure

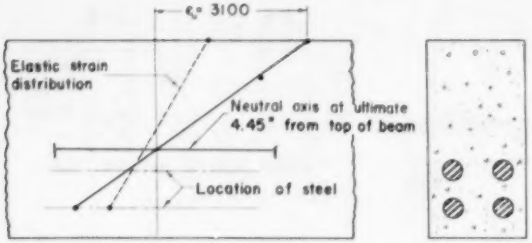

Fig. 11—Typical strain distribution in beam B at ultimate

testing machine with a slow loading rate of about 5000 lb per min. The cylinders were tested in a 440,000-lb hydraulic testing machine which seems to be relatively stiff for 3500-psi concrete. The authors feel that the ultimate strains, from cylinder tests, beyond the ultimate compressive strength, using existing testing techniques, are not as yet reliable enough for beam analysis. However, it is believed that the general shape of the stress-strain curve beyond the ultimate is similar to that in flexure. In the analysis of data the actual ultimate strains obtained in the beam tests were used. The beam tests were conducted in a time interval of about 45 min.

SUMMARY

The validity of the analysis presented is dependent upon the complete stress-strain curve in direct compression of a cylinder corresponding to that of the compression zone in flexure. It has previously been pointed out that the stress-strain relations are similar within the limits of the inconsistencies of concrete.[1]

The approach outlined in this paper would seem to have the following advantages:

1. A realistic continuous function for the relation of stress and strain in flexure, based upon the concrete properties f_c', ϵ_o, and ϵ_u, which should apply to a wider range of concrete properties than those approaches based primarily on f_c'.

2. Accurate predictions of total compressive force, location of neutral axis, centroid of stress block, and ultimate moment, giving a better basic picture of conditions in a beam at failure.

3. The ability to predict mode of failure as demonstrated with the nearly balanced design specimens tested. Fig. 12 indicates that the ultimate moment for concrete failures is not too sensitive to the ultimate strain in the concrete. However, for beams near balanced design the type of failure is more sensitive to the ultimate strain in the concrete. This is indicated in Fig. 6.

4. The realistic treatment of two or more layers of reinforcement steel.

There is a need for a reliable test to determine the ultimate strain, ϵ_u, for concretes with a variety of cements, aggregates, mix proportions, and curing conditions. There is a further need to test the expressions derived in this paper more extensively. If ϵ_u can be accurately determined and the mathematics hold for a wide range of conditions, ultimate design could be confidently conducted without the present dependence upon empirical results.

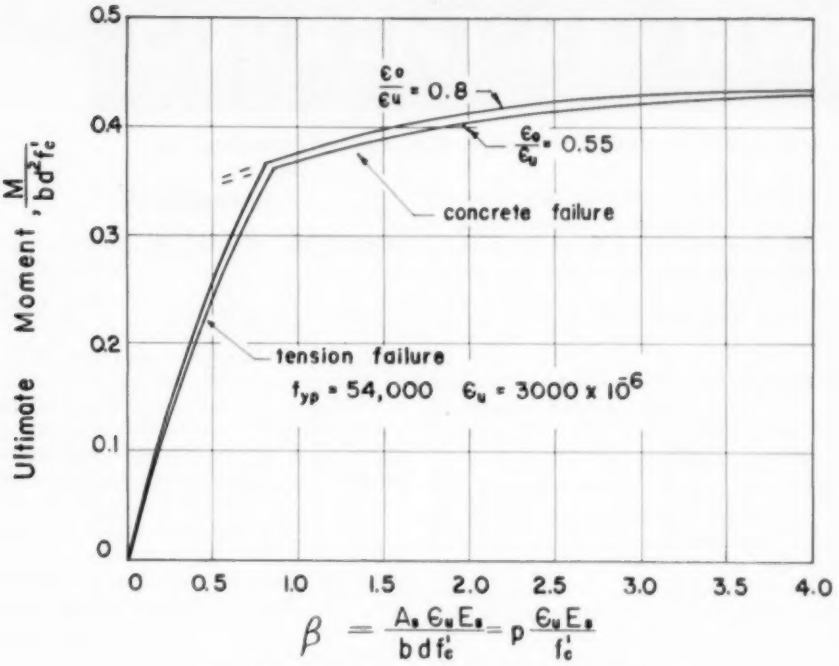

Fig. 12—Effect of ϵ_o/ϵ_u on ultimate strength

ACKNOWLEDGMENTS

This investigation was sponsored by research grants from the George Abel Memorial Fund and the University Research Council of the University of Nebraska.

Tests were performed in the Department of Engineering Mechanics, College of Engineering and Architecture, University of Nebraska, under the direction and with the encouragement of E. J. Marmo, chairman.

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2. Cox, Kenneth C., "Test of Reinforced Concrete Beams with Recommendations for Attaining Balanced Design," ACI JOURNAL, Sept. 1941, *Proc.* V. 48, pp. 65-80.

3. Whitney, C. S., discussion of a paper by V. P. Jensen, "The Plasticity Ratio of Concrete and Its Effect on Ultimate Strength of Beams," ACI JOURNAL, Nov. 1943, Supplement, *Proc.* V. 39, pp. 584-2 to 584-6.

For such discussion of this paper as may develop please see June 1957 JOURNAL.

CURRENT REVIEWS

of Significant Contributions in Foreign and Domestic Publications

Construction

Construction of Castle Donington power station

The Engineer (London), V. 201, No. 5235, May 25, 1956, pp. 557-561

Reviewed by ARON L. MIRSKY

This completely new station on the River Trent, with six 100 MW generating sets, presents some interesting structural features in both steel and concrete. The foundation blocks for the turbo-alternator, which weigh 350 tons each, are of prestressed concrete; the raft foundation in the turbine area, 580 x 80 ft x 17.5 ft thick, was Colcreted.

Radio transmission towers for the state postal service (in Dutch)

H. J. J. ENGEL, *Cement* (Amsterdam), V. 7, No. 11-12, Dec. 9, 1955, pp. 261-271

Reviewed by JOHN W. T. VAN ERP

These towers are part of the broadcasting and television network of the state. They have a set of four platforms at 200-260 ft elevation which have an unobstructed view in all directions, with operating rooms for equipment at the same levels. This determined their shape. They have been constructed by the slip-form method using a cylindrical shell 8 in. thick, which is about the minimum required thickness for adequate steel coverage (2.4 in. inside and outside) and a double cage of reinforcement. The slip-forms simultaneously form the exterior shell and also the interior partitions for elevator shaft and staircase. To construct intermediate floors and cantilevered slabs, the necessary amount of dowel steel was incorporated in the slip-form shell and bent out later. For this dowel steel soft plain bars are used; all other steel is high-tensile strength deformed bars. An additional advantage of this steel in the thin members of slip-form construction is that it is not easily bent and stays straight within narrow limits. In connection with the design of these towers a research program on wind loads at great height was conducted.

Television tower of Stuttgart (Der Stuttgarter Fernsehturm)

FRITZ LEONHARDT, *Beton- und Stahlbetonbau* (Berlin), V. 51, No. 4, Apr. 1956, pp. 73-85

Reviewed by VALDIS LAPSINS

A reinforced concrete structure to be used as a television transmitter tower and an observation tower for public use. The entire structure consists of the following component parts: (1) management, business, and restaurant building at the tower base; (2) round reinforced concrete tower, 161 m high, containing a shaft for two elevators; (3) tower head with its greatest diameter of 15.10 m at an elevation of 138 m above ground level, containing four floors with facilities for transmitter, restaurant, and an open observation deck; (4) steel frame antenna tower, 50 m high, making the total height of the tower 211 m.

The outside diameter of the reinforced concrete tower at the base is 10.8 m, with 60 cm thickness of walls. Ten m above ground wall thickness decreases to 30 cm, and at the tower head 138 m above ground the thickness is as little as 19 cm with a tower diameter of 5.04 m.

Description of the structure, many details and design considerations are presented. Erection features and some observations as to the behavior of the tower are described in *Beton- und Stahlbetonbau*, V. 51, No. 5, May 1956, pp. 104-111.

A part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 28, No. 6, Dec. 1956, *Proceedings* V. 53. Address 18263 W. McNichols Rd., Detroit 19, Mich. Where the English title only is given in a review, the book or article reviewed is in English. If it is followed by a foreign title the work reviewed is in that language. In those cases where the foreign title cannot conveniently be set in type or is not available, the language of the original article is indicated in parentheses following the English title. Copies of articles or books reviewed are **not** available through ACI. Available addresses of publishers are listed in the June "Current Reviews" each year. In most cases ACI can furnish addresses of publications added later.

For those members that cut apart this section for pasting on cards for card indexes, a limited number of complimentary reprints of the "Current Reviews" section are available from ACI headquarters on request.

Dams

Arch dams: Design and construction of Ross Dam

C. E. Shevling and L. R. Scrivner, *Proceedings*, ASCE, V. 82, PO4, Aug. 1956, pp. 1045-1 to 1045-18

A brief description of design and construction details of Ross Dam in Washington.

Design

Special kind of shells of translation with two vertical planes of symmetry

P. Csonka, *Acta Technica Academiae Scientarium Hungaricae* (Budapest), V. 11, No. 1-2, 1955, pp. 231-240
Reviewed by Gunhard Aestius Oravas

Amplifies author's previous work (see "Current Reviews," ACI Journal, Apr. 1956, *Proc.* V. 52, p. 897) by demonstrating that his method of choosing the proper generating curves for shells of translation with two vertical planes of symmetry to the rectangular base projection results in a very simple analysis of membrane forces for various types of loads normal to the base, which occur often in the practical shell design.

Author expresses the loading and the stress function in terms of finite polynomials and obtains the unknown parameters by a simple calculation. A numerical example illustrates the procedure.

Bending theory of translational surfaces and their application in arena structures (Biegetheorie der Translationsflächen und ihre Anwendung im Hallenbau)

Konrad Hruban, *Acta Technica Academiae Scientarium Hungaricae* (Budapest), V. 7, No. 3-4, 1953, pp. 425-464
Reviewed by Gunhard Aestius Oravas

This paper analyzes shallow shells of translation according to the theory set up by Vlassov. The membrane stresses and boundary disturbance effect of translational shells are treated in a practical manner for various types of shells. Discussions of application of the theory in practical design follows. Construction details and cost analysis of structures which were executed by the author in Czechoslovakia are given at the end of the paper with numerous photographs describing field construction.

Additions to the method of analysis of multiple-girder bridges (Compléments à la méthode de calcul des ponts à poutres multiples)

C. Massonnet, reprinted by University of Liege, Belgium, from *Annales des Travaux Publics de Belgique* (Brussels), Oct. 1954, 68 pp.
Reviewed by Aron L. Mirsky

Guyon's method of analyzing bridges with multiple stringers and cross-beams is extended by including torsional rigidity of the stringers and effect of edge beams of higher stiffness. Method is stated to involve (1) substituting for the actual structure a continuous structure (*i.e.*, an anisotropic plate) which has the same average flexural and torsional rigidities as the actual structure but which is analyzable by theoretically exact methods (the error involved in this substitution has been shown to be very small), and (2) analyzing in approximate fashion the effect of the transverse division of the loads. Latter is based on sinusoidally-distributed loads, but method is said to apply to other types of loadings as well, especially if additional terms of a Fourier series are used.

Well over two-thirds of the paper (pp. 21-67) is devoted to tables intended to sweeten the lot of the designer faced with the task of calculating the transverse division of loads among the various members. Author accomplishes this by use of two parameters. Θ, the factor of grid rigidity (*entretoisement*) and α, the factor of torsional rigidity.

(See also the following review.)

Thoughts on the Guyon-Massonnet method of analysis for simply supported grid frameworks and extension of this method to arbitrary systems (Betrachtungen zum Berechnungsverfahren von Guyon-Massonnet fuer freiaufliegende Traegerroste und Erweiterung dieses Verfahrens auf beliebige Systeme)

Konrad Sattler, *Der Bauingenieur* (Berlin), V. 30, No. 3, Mar. 1955, pp. 77-89
Reviewed by Aron L. Mirsky

Guyon and Massonnet (see preceding review) developed analysis of multi-girder structures under action of concentrated load using an idealized anisotropic plate of constant thickness. Author shows method yields results in good correspondence with

model test results, for multi-girder multiple-span (continuous but simply supported) structures. Guyon-Massonnet values are presented in graphical form as aids to designers.

Discussion appears in the July 1955 issue, p. 276.

Numerical analysis of rigid frames in space (in Japanese)

SHUN-YA YOSHIDA, *Transactions*, Japan Society of Civil Engineers, No. 33, Apr. 1956, pp. 1-6
AUTHOR'S SUMMARY

There are several cases in which some effects of torsional moments may not be neglected. In these cases the structures must be analyzed under their actual states, *i.e.*, three-dimensionally. But, in general, it is so troublesome to analyze the structures three-dimensionally that such analyses are disliked by many engineers.

In this paper, it is shown that a numerical analysis derived from the fundamental conceptions of the moment distribution method is a convenient tool to analyze rigid frames in space three-dimensionally.

Consideration on safety of structures by plastic and stochastic theory (in Japanese)

I. KONISHI and M. SHINOZUKA, *Transactions*, Japan Society of Civil Engineers, No. 33, Apr. 1956, pp. 7-11
AUTHORS' SUMMARY

For a beam subject to a single rolling load W, from the concept of limit design, there are two types of failures; the one is static collapse and another is incremental collapse. The theoretical method to determine the probability that either static collapse or incremental collapse occurs is represented.

Highway bridge slabs (Fahrbahnplatten von Strassenbrücken)

Bulletin No. 106, Deutscher Ausschuss für Stahlbeton (Berlin), 3rd Edition, 1956, 162 pp., 24 DM

Presents (80 pp.) plate theory for bridge slabs, skew or rectangular, supported one, two, three, or four sides, simple or continuous, and reinforced by rectangular or skew grids. Second section of the book gives tables for numerical solution of various cases. Application of the methods is demonstrated by numerical examples.

Calculation of callotte shells over rectangular bases

P. CSONKA *Acta Technica Academiae Scientarium Hungaricae* (Budapest), V. 11, No. 3-4, 1955, pp. 428-440
Reviewed by GUNHARD AESTIUS ORAVAS

Author develops a method of membrane analysis for callotte shells (generalized dome type of shells) over rectangular base and two vertical planes of symmetry by developing the stress function and normal loading to the base into twin power series. Then the divergence of the chosen loading function, which corresponds to the approximate stress function, is minimized from the actual loading system by the application of mean root square error.

This method converts a difficult problem into relatively simple routine and is of great practical significance to the shell designer.

Callotte shells over rectangular base

P. CSONKA *Acta Technica Academiae Scientaium Hungaricae* (Budapest), V. 13, No. 1-2, 1955, pp. 149-164
Reviewed by GUNHARD AESTIUS ORAVAS

Generalizes on previous paper (see above review) by the author in treating the membrane theory of callotte shells over rectangular base and with boundary arches of varying crown heights.

The stress function and normal loading to the base are assumed as polynomials and the method of collocation is used to make the loading function corresponding to the assumed stress function agree with the actual loading at a number of specified points.

A comparison with the method of the author's previous paper is made in a numerical example. The accuracy of the method is for practical design more than sufficient.

Rigorous investigation of composite type trusses (Strenge Untersuchung des Verbundfachwerktraegers)

GEORG NITSIOTAS, *Der Bauingenieur* (Berlin), V. 30, No. 7, July 1955, pp. 240-244
Reviewed by ARON L. MIRSKY

After investigating effect of creep on distribution of forces in symmetrically-reinforced concrete slabs, with and without prestress, author deduces and then integrates the differential equations of statically indeterminate composite type trusses with members of either variable or constant rigidity and with the concrete slabs forming one of the chords.

Contribution to practical determination of maximum axial forces in piles supporting foundation slabs (in Serbian)

V. Kostic, *Nale Gradevinarstvo* (Belgrade), V. 10, No. 4, Apr. 1956, pp. 493-498

Reviewed by J. J. Polivka

Method of influence lines for each individual pile is used. Culmann's method is used for batter piles. It is assumed that piles having the same inclination carry equal loads. Axial forces are proportional to the statical moment about the neutral axis of each pile. Numerical examples are presented.

Buckling of coplanar hinged arches (Le flambement des arcs articules a plan moyen)

Pierre Blaise, *Annales des Ponts et Chaussees* (Paris), V. 126, No. 1, Jan.-Feb. 1956, pp. 1-38

Reviewed by Aron L. Mirsky

Having studied hyperstatic arches in a previous paper (*Annales*, V. 124, No. 6, Nov.-Dec. 1954, pp. 635-669; "Current Reviews," ACI Journal, Jan. 1956, *Proc.* V. 52, p. 594), author here derives expressions for arches with one, two or three hinges (articulations). As numerical examples, parabolic arch investigated in 1954 paper is modified by provision of hinges (1) at crown, (2) at both springings, and (3) at all three of these points.

Influence line ordinates at the 10th points of continuous beams—V. 3 (Zehnteilige Einflusslinien fuer durchlaufende Traeger—Band 3)

Georg Anger, Wilhelm Ernst & Sohn, Berlin, 1955, 247 pp., 27 DM

Reviewed by Fritz Kramrisch

This work, which can be used independently from the other two volumes, not only deals with continuous beams having different span ratios, but also includes single-span beams (simply supported, fixed at one end, and fixed at both ends). Beams over three supports are treated for span ratios of 1:1 to 1:2.5 in increments of 0.1. For all beams with more than three supports all interior bays are assumed to be alike, and the two exterior bays are equal too, but the ratio between the spans of the exterior and the interior bays varies. For beams over three bays the span ratio varies from 1:0.5:1 to 1:2:1 including ratios of 1:1.28:1 and 1:1.35:1 frequently used in the United States. Beams over four bays are treated for span ratios from 1:0.5:0.5:1 to 1:2:2:1 and beams over five bays for ratios of 1:0.8:0.8:0.8:1 to 1:2:2:2:1. All variations in the span ratios are worked out for increments of 0.1.

The tables furnish the ordinates of the influence lines for moment and shear, the maximum moments due to the loading of certain bays, and the moments due to a uniformly distributed load.

In addition to the ordinates at the 10th points of every bay, the values at the ¼- and ⅓-points are furnished.

Furthermore there are tables giving the influence of cantilever (end) moments for all numbers and ratios of spans, and influence lines for an interior bay of a continuous beam with infinitely many bays.

This reliable tabular work can be used with only little knowledge of the German language by any engineer or designer who has a fair understanding of the setup of such tables.

The introduction, besides explaining the tables, gives derivations, direct formulas for certain types of loadings, and applications to variable moments of inertia. The accompanying text is concise but unfortunately not too easily understood for someone with limited knowledge of German, and in the setup of the formulas is somewhat different from that used in this country; but an experienced engineer should be able to put the tables to practical use without much help from the introduction.

Examples of simplified computation of the loading terms in the elasticity equations

Otto Braun, *Journal*, Boston Society of Civil Engineers, V. 43, No. 2, Apr. 1956, pp. 128-141

Reviewed by Aron L. Mirsky

Principle enunciated by author in 1952 (*Beton- und Stahlbetonbau*, V. 47, No. 1, Jan. 1952), that "It is permissible to let the external loading operate on any possible primary system (*i.e.*, to use any possible condition of equilibrium for the external loading), no matter what primary system was used when it was subjected to the redundants," is used for the solution of various types of two- and three-dimensional redundant structural systems.

Composite prefabricated reinforced concrete structures (Skiver opbygget af elementer)

H. O. Sonne Andersen, *Ingenigren* (Copenhagen), V. 65, No. 30, July 28, 1956, pp. 598-605
Reviewed by Jesper Strandgard

The introduction of prefabricated elements has caused some new problems for the statical calculations of structures. These problems relate mainly to the joints between elements. The most important questions are: (1) determination of the strength of the joints; (2) determination of the internal forces in the joints; (3) determination of the deflections caused by the deformations in the joints.

The strength of the joints is dealt with in section I. For joints with straight edges the strength can be determined by Coulomb's law, in which the coefficient of friction can be taken as 0.75, while the cohesion is rather dubious because of shrinkage. Employing keyed joints, it is more difficult to compute the strength. Applying Coulomb's law to a concrete test cylinder, we obtain the condition for failure. The failure strength of the key is influenced by the angle v and the cylinder strength of the concrete, which is shown in the equations and the table following them. The keys prevent mutual sliding of the elements, and they provide a new formal coefficient of friction in the joints.

The internal forces in a composite structure are usually statically indeterminable, even when the structure is of statically determinate nature. Employing rectangular elements, each element may have six unknown items (forces or coordinates) of which only three can be determined by the equations of equilibrium.

In section IIIA is discussed a simple beam with straight edges in the joints. More important is a similar beam with keyed joints. The first part of section IIIB is a computation of this beam, as far as possible without assuming anything about the deformations.

A calculation of the deformations in the joints is made in the second part of section III to design the cogs. The assumptions should be verified, before the computations can be accepted as final. However, it is made evident that more simplified calculations are generally not justifiable, when they do not take the deformations into account. The most frequent assumption of distributing the shear force equally over the joints is usually erroneous. The calculation of deformations can be avoided in very simple cases only, one of which is dealt with in section IIID.

Materials

Study of hardening (dielectric behavior of several Spanish cements) (in Spanish)

J. M. Tobio, *Bulletin* No. 158, Instituto Técnico de la Construcción y del Cemento, Madrid, June 1955, 43 pp.
Reviewed by E. Rosenblueth

Includes a clear presentation of the technique and theory of measuring admittance, resistance, and capacitance of cement pastes under high-frequency currents with very stable equipment. Running tests at a variety of frequencies, author finds range of frequencies adequate for optimum definition of admittance-time curve. Curves of this type are presented for 20 Spanish brands of portland cement, several samples of gypsum and gypsum-cement mixtures, and the shape of admittance-type curves is quantitatively correlated with setting and hardening times of cement pastes and mortars and qualitatively with compressive strength at various ages. It is shown that admittance at high frequency is not too greatly influenced by temperature variations. Several applications are suggested, among which superseding the Vicat-needle method seems the only promising one, for apparently, dielectric measurements lead to more consistent results. Summaries in Spanish, French, English, and German.

Prestressed Concrete

Some studies on the behavior of prestressed concrete columns

A. M. Ozell and A. M. Jernigan, *Technical Progress Report* No. 3, Florida Engineering and Industrial Experiment Station, University of Florida, Gainesville, July 1956, 12 pp.

Presents the results of tests on 11 approximately half-size column specimens. The tests reported show some advantage in the use of prestressed columns, but are inconclusive in this regard. A formula is developed for the prediction of the ultimate actual load capacity of columns with L/d ratios between 20 and 30.

Properties of Concrete

Freezing and thawing tests on green concrete

POUL NERENST and NIELS MUNK PLUM, *Reprint* No. 73, Danish National Institute of Building Research, 1956, 44 pp.

AUTHORS' SUMMARY

Experiments with freezing of concrete at early ages have been carried out by the Danish National Institute of Building Research and the Danish State Testing Laboratory in the years 1949 and 1950. It was the aim to duplicate actual weather conditions in Denmark in order to determine how early concrete *without entrained air* may be exposed to freezing without causing a permanent damage to concrete. Test results are presented, but due to difficulties of controlling the degree of saturation and other pertinent factors the results are rather confusing. Later recommendations issued by this Institute have been based upon theoretical computations of freezing resistance of concrete *with entrained air*. As air entrainment has not been adopted by all countries the data obtained may be of interest in spite of the lacking of clear-cut conclusions. Difficulties of discrimination between influences from frost damage, delayed hardening due to low temperature and autogeneous healing by this type of investigation are briefly discussed.

Structural Research

New method to determine tensile strength of concrete (Ein neues Verfahren zur Bestimmung der Betonzugfestigkeit)

COELESTIN ZELGER, *Beton- und Stahlbetonbau* (Berlin). V. 51, No. 6, June 1956, pp. 139-140

Reviewed by VALDIS LAPSINS

Same testing equipment and test specimens (cylinders) are used for either measuring compressive or tensile strength of concrete. The main difference lies in the manner in which the tension specimen is placed in the testing machine. It lies flat. Loads applied by the testing apparatus are transmitted to the test cylinder sides by means of load distributing plates and small (not wider than 1/10 of the cylinder diameter) cutting blades which activate the splitting of the specimen and which are placed at opposite sides of the specimen.

General

Nuclear explosions and resistance of materials (Explosions nucleaires et resistance des materiaux)

R. PAOLI, *Annales des Ponts et Chaussees* (Paris). V. 126, No. 2, Mar.-Apr. 1956, pp. 127-162

Reviewed by ARON L. MIRSKY

Paper, a combination of comment on, justification of, and appendix to the French Minister of the Interior's bulletin of Jan. 25, 1955, on protection against the effects of atomic weapons, covers in considerable detail the behavior of blast waves and the action of materials under dynamic loading, especially atomic blasts. A brief numerical example concerns a square reinforced concrete shelter slab.

While much of the material parallels that in the prize-winning ACI paper by C. S. Whitney, B. G. Anderson, and E. Cohen (ACI JOURNAL, Mar. 1955, *Proc.* V. 51, pp. 589-683), it is by no means identical.

Road to modern building in reinforced concrete (Der Weg zum Neuzeitlichen Stahlbetonbau)

HERMANN BAY, *Beton- und Stahlbetonbau* (Berlin), V. 51, No. 6, June 1956, pp. 127-135

Reviewed by VALDIS LAPSINS

Historical approach to the science of theory and practice in building in general with given emphasis to building in reinforced concrete. Coordination of knowledge and ideas of engineers and architects promoted for a more fully accepted position in the building field and strive to realization of perfection.

Practical estimates of materials in reinforced concrete (in Spanish)

M. BOUSO, *Bulletin* No. 161, Instituto Técnico del la Construcción y del Cemento, Madrid, July 1955, 48 pp.

Reviewed by E. ROSENBLUETH

Systematic set of rules, data sheets, graphs, tables, and formulas with worked examples for estimating length and weight of reinforcement, as well as concrete and excavation volumes. Presentation is outstanding, particularly in clarity of exposition. All tables and graphs are in metric system and applicability is restricted to smooth bars of diameters usual in Spain. Summaries in Spanish, French, English, and German.